Handout on Disneyland

Suppose that a firm faces two types of customers, highs and lows, with two different demand curves, $p^H(q)$ and $p^L(q)$, where $p^H(q) \geq p^L(q)$.

Note that the total surplus for a type j customer from quantity Q is given by $TS^j(Q) = \int_0^Q p^j(q)dq$. The net surplus from bundle (Q,P) is $TS^j(Q) - P$.

Disneyland, who faces a marginal cost of zero, wants to price discriminate by offering two bundles: (Q_1,P_1) and (Q_2,P_2). Disneyland has in mind targeting the first bundle to the low types and the second bundle to the high types. What conditions guarantee that the customers will buy the bundle targeted at them?

> **Participation constraints** (ensure customers get positive utility from their bundle)
>
> (PL) $TS^L(Q_1) \geq P_1$
>
> (PH) $TS^H(Q_2) \geq P_2$
>
> **Self-selection constraints** (ensure customers prefer their bundle to the bundle targeted at the other guy)
>
> (SSL) $TS^L(Q_1) - P_1 \geq TS^L(Q_2) - P_2$
>
> (SSH) $TS^H(Q_2) - P_2 \geq TS^H(Q_1) - P_1$

Now, the firm wants to maximize total revenue, as follows:

$$\max_{(Q_1,P_1),(Q_2,P_2)} n_L P_1 + n_H P_2$$

subject to: (PL), (PH), (SSL), (SSH)

The way we proceed is as follows. We argue that whenever (SSH) and (PL) are satisfied, then (PH) and (SSL) are. I'll leave the proof of this until the end, and you're not responsible for it. Intuitively, it will turn out that the low type will find the "high" bundle is very unattractive, so we won't have to worry about (SSL). Further, the high type will have to receive strictly positive net surplus from the high bundle to avoid having the high type deviate and choose the low bundle, so that SSH satisfied will *imply* that (PH) is satisfied.

By these arguments, the firm's problem simplifies to:

$$\max_{(Q_1,P_1),(Q_2,P_2)} n_L P_1 + n_H P_2$$

subject to: (PL), (SSH)

We could solve this as a Lagrangian (if you want, try it), but notice something: for a given set of quantities we want to charge the highest prices we can subject to the constraints. What's the highest price P_1 such that (PL) is satisfied? Well, exactly $P_1 = TS^L(Q_1)$. What's the highest price P_2 such that (SSH) is satisfied? Well, set $TS^H(Q_2) - P_2 = TS^H(Q_1)$