14.121 Waiver Exam -- Sept. 1996

Instructions: Relax. Your score on this exam will not be used for any purposes other than deciding whether or not you need to take 14.121 this fall.

Answer all questions. If you need to make *any* assumptions beyond those given in the problem, state them clearly and state why you need them. You may invoke mathematical results without proving them, so long as you are clear about which result you are using and why it applies.

Question 1 - 30 minutes

Consider a town filled with utility-maximizing citizens, all of whom have preferences which satisfy the Von Neumann-Morgenstern axioms, which can be represented with a utility function $u(w)$, where the function $u(\cdot)$ is the same for all agents. Assume that this utility function is differentiable as many times as you like, and further it is strictly increasing and concave.

Every individual citizen has some initial wealth as well as a car which must be parked at metered parking spaces each day. The cost of the meter is given by m. If the meter is not paid, it indicates "violation." The town hires police who patrol the area, and with probability p the violation is spotted and a fine f is levied. If the police do not spot the violation, the individual pays nothing.

Consider the problem faced by an individual who maximizes their utility for the day by choosing whether or not to feed the meter (for non-Americans, "feed" is slang for "put money into"). Let Δ denote the returns to the agent from feeding the meter, that is, the difference between the agent's expected utility from feeding the meter and their expected utility from parking illegally.

(i) Suppose that the town council considers funding more police, which increases p, versus raising the fine f. Compute the elasticity of Δ with respect to p, and compare it to the elasticity of Δ with respect to f. To which policy change are the citizens more responsive?

(ii) Now suppose that citizens of this town differ according to their initial wealth, w_0, on the range $[\underline{w}, \overline{w}]$. *For the rest of Question 1,* suppose the parameters of the problem are such that Δ is decreasing in initial wealth, and that we observe some citizens feeding the meters and others parking illegally. (a) Write a simple expression which determines which group of people feeds meters and which group parks illegally. (b) What can we infer about the relationship between the price of the meter, m, and the expected value of the fine from parking illegally? Sketch a proof of your answer.

(iii) Qualitatively, how does the set of people who feed the meters change with *f*, *m* and *p*? Prove your answer and interpret your results.

(iv) Now suppose that the police department gets to keep the revenue from parking fines and meters. Further, the town council sets *m* and *f*, that is, *m* and *f* are exogenous to the police department. However, the police deparment can choose how carefully to check the meters (i.e. it determines *p*), at some cost $c(p)$. The police department simply maximizes revenue from parking. Suppose that the city council votes to increase *f*. (a) Derive an expression which shows how a small change in *f* will affect the police department's revenue. Is a small increase unambiguously good for the department? (b) Will an increase in *f* always lead the police department to check for violations more often? If so, sketch a proof, and if not, describe the competing effects.

<u>Question 2 - 30 minutes</u>

Instructions: *In problems (i) and (ii) below, you are asked whether you can draw certain conclusions from the given assumptions. If so, sketch a proof which indicates any assumptions of the problem you use in each step of your answer. If the answer is "it depends," you* ***must*** *give an example of a set of assumptions under which the statement is true, as well as an example of a set of assumptions under which the statement may be false. In all cases, interpret your answers.*

Consider a profit-maximizing fast-food restaurant which invests money in equipment (*k*), at a rental price of capital given by *r*, and hires workers (*l*). The cost *per hour* from hiring *l* hours of labor is $W(l)$, which is nondecreasing in *l*. The firm produces fast food according to the function $F(k,l)$, and sells this output at a constant price *p*. The equipment *k* is fixed in the short run. Suppose that we are currently in a long-run equilibrium, and the restaurant is currently using l_0 labor at a cost of $W(l_0)$ = \$4/hr.

Assume throughout the problem that (i) *F* is nondecreasing, (ii) the marginal productivity of labor is diminishing for all values of *k*, and (iii) there is a unique, interior optimal choice of equipment and labor for all relevant parameter values. If you need *any* additional assumptions, you must state them.

Now suppose that Congress votes to institute a minimum wage of $w^{\min}$=\$5.

(i) Consider the firm's short run problem of choosing a new level of labor, l_s. Can you conclude that $l_s \leq l_0$? [Your analysis here should be partial equilibrium; do not worry about general equilibrium feedbacks.]

Now suppose that the firm has been producing in a long-run equilibrium with the minimum wage in place, using labor l_0' at wage $w^{\min}$=\$5. Then Congress raises the minimum wage again, to \$5.25.

(ii) Can you conclude that the firm will adjust its demand for labor more in the long run than in the short run? That is, let the firm's new short run labor labor choice be l_s', and the new long-run labor choice be L. Then, can we conclude that $L \leq l_s' \leq l_0'$?

(iii) Finally, let us change the problem a little bit. Forget all of our previous assumptions about the cost of labor, and suppose instead that the firm's *total* cost (not per hour cost) of hiring l hours of labor is given by a function $C(l;t)$. Further, suppose that the marginal cost of labor ($C_l(l;t)$) is nondecreasing in t, but you make no further assumptions about C. In response to an increase in t, what happens to the firm's short run choice of l? Compare your answer to part (i), and explain similarities and differences.

Question 3 - 30 minutes

You are called on by the President to provide an estimate of the increase in the cost of living in MIT economics dept. graduate students. You observed that in 1995 and 1996, graduate students bought only three items: bagels, coffee, and pizza. The "representative" graduate student has preferences which do not change over time, and the student is assumed to maximize utility, exhausting all of their income on these three items. Each of these items can be freely disposed, and all items are available at only one price, which you observe.

The graduate student had the following consumption patterns:

1995:

400 bagels at $1 each, 400 cups of coffee at $.75 each, and 350 pizzas at $7.50 each.

1996:

375 bagels at $1.20 each, 400 cups of coffee at $1 each, and 336 pizzas at $7.50 each.

(i) Can you say in which year the graduate student was better off?

(ii) Give two alternative measures of the change in the consumer's well-being ("the cost of living") from 1995 to 1996 (i.e., Laspeyres and Paasche price indices; if you can't remember the labels, just describe two sensible measures) based on this data.

(iii) Suppose you knew the the graduate student's utility function, $u(B,C,P)$. How would you calculate an ideal measure of inflation, which reflects the change in the consumer's well-being between 1995 and 1996? Precisely state the interpretation of the measure you provide.

(iv) Prove any inequality restrictions which exist between the ideal measure of the cost of living you proposed in part (iii) and your measures from part (ii). Be precise.