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# The Novelty of Innovation: Competition, Disruption, and Antitrust Policy

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**Abstract.** We develop a model to capture the novelty of innovation and explore what it means for the nature of market competition and quality of innovations. An innovator decides not only whether to innovate but how boldly to innovate, where the more novel is the innovation—the more different it is from what has come before—the more uncertain is the outcome. We show in this environment that a variant of the Arrow replacement effect holds in that new entrants pursue more innovative technologies than do incumbents. Despite this, we show that the new entrant is less likely to disrupt an incumbent than the incumbent is to disrupt itself, and less likely to fail in the market. We extend the model to allow the incumbent to acquire the entrant postinnovation and show that this reverses the Arrow effect. The prospect of acquisition makes innovation more profitable but simultaneously suppresses the novelty of innovation as the entrant seeks to maximize her value to the incumbent. This reversal suggests a positive role for a strict antitrust policy that spurs entrepreneurial firms to innovate boldly.

**History:** Accepted by Joshua Gans, business strategy.

**Keywords:** game theory • innovation • microeconomic behavior • market structure • firm strategy • market performance

## 1. Introduction

Innovation is not just a matter of doing things better. It is also a matter of doing things differently. Sometimes these differences are small, sometimes they are enormous, with ideas and technologies that depart radically from those that came before.

This presents budding entrepreneurs with an additional choice. They must decide how boldly to innovate and not just whether to innovate. What technology should they pursue in their efforts at change? The more radical an innovation is, the more likely they achieve a radical breakthrough, but also the more likely they suffer a radical failure.

The choice of technology with which to innovate also matters for the nature of market competition. The further a technology is from the mainstream—from an industry's dominant design—the more different is the set of consumers it serves and, therefore, the softer is the competition with the incumbent firm.

The market entry strategy of a new firm must balance these dual considerations of technology and market competition. In this paper we explore and characterize the incentives to innovate in such an environment, focusing on the novelty and riskiness of the innovations undertaken. This allows us to see not only whether there is innovation in a market, but how novel it is, and what that means for market efficiency,

both in terms of the technological improvements it delivers and in the degree of market competition it generates. We then trace through the implications for firm strategy in terms of mergers and acquisitions.

To get at these issues, we introduce a novel model of technological uncertainty in which the possibilities for innovation are rich and fall along a continuum, as also do the outcomes. This covers the full gamut of innovation, from incremental steps to bold innovations, from breakthrough outcomes to memorable flops, and everything in between.<sup>1</sup> In the model, new market entrants choose how radical they wish to be. A critical feature of the modeling technology is that the uncertainty over the outcome of the innovation increases in the radicalness of the experiment. This captures the idea that the more we move from what is known—the greater the distance from existing technology—the more uncertain is the outcome.<sup>2</sup> At the same time, the more novel is the innovation, the more distant it is from the mainstream taste of consumers and the lower is the intensity of market competition with the incumbent firm.

We apply this model to a classic question: Is innovation more likely to come from an independent entrepreneur or an established firm? We differ from classic accounts in that we examine the question not in terms of the amount, or intensity, of innovation but in terms

of the novelty of innovation. Nevertheless, we show that an amended Arrow replacement effect continues to hold in that the incumbent always innovates less radically and more incrementally than does the entrepreneur (Arrow 1962). We refer to this as the *spatial Arrow effect* as it relates to the differences between innovations rather than the intensity of innovation.<sup>3</sup>

Despite the similarity, the logic for our result differs from Arrow's. Arrow's argument is that the incumbent has less incentive to innovate as it already has the dominant product and, so, only receives the marginal gain from an innovation. In our setting, this logic pushes the incumbent's innovation away from its existing product and toward more innovation. The more novel is the new product, the less it cannibalizes the existing product, and the more the incumbent's portfolio of products appeals to the broad range of customer tastes. In contrast, as Arrow pointed out, the independent entrepreneur has the added incentive of stealing market share from—of replacing—the incumbent. The classic Arrowian logic suggests, therefore, that the entrepreneur will stay tight to the incumbent as by doing so it can potentially gain more of the market.

To see why the entrepreneur nevertheless innovates more, we must draw in another classic intuition, that of Hotelling's (1929) spatial competition. Because a more novel innovation is more distant also in terms of market competition, the entrepreneur has the incentive to differentiate to soften market competition. The incumbent, being able to coordinate pricing across products, does not have this incentive. We show that the competition-softening incentive of the entrepreneur dominates the market-spanning incentive of the incumbent, and the spatial Arrow effect follows. In this way, the combination of classic insights from spatial differentiation and price competition yield new insights into classic questions of innovation.

These differences lead to subtle but important implications about what it means to disrupt. Disruption depends not only on the entrant's technology. It also depends on how distant the entrant is in the technology space, for consumers may still prefer an incumbent product if the new technology, even if it is of higher quality, does not serve their needs as well. We show that this leads to an inversion of the standard logic of who disrupts and when it happens, of when a new product drives a competitor from the market. We show that while the independent entrepreneur engages in a bolder experiment, and thus has a higher probability of a breakthrough outcome, the entrepreneur disrupts the incumbent product with smaller probability than does the incumbent disrupt itself. That is to say, the probability that the incumbent product remains in the market is lower when the incumbent itself innovates and innovates more incrementally. Self-disruption in

this way is less dramatic—the innovation may only be a marginal improvement—yet given its proximity in the technology space, it is enough to render the incumbent product obsolete. In contrast, the entrepreneur may possess a markedly better product yet still not drive the incumbent from the market. This pattern provides a novel perspective on why apparently disruptive innovations do not necessarily drive incumbent firms from the market (Gans 2016).

We then use the model to explore questions about firm strategy and antitrust policy in innovative industries. These questions have risen to prominence in recent years in academic debate and, even more so, in the public domain. The increase in market concentration in the United States and a corresponding decline in investment has led many to question whether accommodative antitrust policy has led to the suppression of innovation.<sup>4</sup>

We investigate these questions by allowing the incumbent firm to take over, or merge, with the entrepreneurial upstart. Such a move is anticompetitive and, thus, desirable to the firms. The question of our interest, then, is not whether the merger occurs, but what effect its prospect has on the *ex ante* incentives of an entrepreneur to innovate and the novelty of the innovation the entrepreneur chooses.

We show that the prospect of being acquired by the incumbent causes the entrepreneur to moderate in its innovation, choosing a more incremental innovation that is closer in technological space to the incumbent. In fact, we show that this effect is sufficiently strong that it overturns the spatial Arrow effect when the entrepreneur has enough bargaining power. In this case, the prospect of merging induces the entrepreneur to choose a more incremental innovation than would the incumbent itself.

The spatial Arrow effect itself suggests that the entrant can add the most value to the incumbent by innovating more boldly, as distancing itself from the incumbent would allow the merged entity to span more of the market, imitating what the incumbent would do itself. That much is true. But a second force drives in the opposite direction. By moving toward the incumbent, the entrepreneur poses a greater competitive threat and, thus, ensures that its removal is of even greater value to the incumbent. This pushes the entrepreneur toward rent-seeking behavior, to act against efficiency because doing so improves its bargaining leverage over the incumbent. We show that this force dominates, overturning the spatial Arrow effect and undermining innovation in the market.

In Silicon Valley, as elsewhere in the world, the goal of many entrepreneurs is not an initial public offering (IPO), rather it is to be acquired by an existing incumbent firm, such as Google, Microsoft, and the like. This practice is often pointed to as evidence that

innovation is flourishing. Even though the market power of the large tech firms may appear overwhelming, the argument goes, acquisitions provide a viable and valuable channel for innovation. Our result shows the dark side of this practice. A lax antitrust policy that allows dominant firms to gobble up emerging competitors certainly does provide a spur to innovation, but it does so in way that pushes entrepreneurs toward incremental innovations. Thus, the market, and society at large, miss out on the bold and breakthrough innovations that otherwise would emerge if competition were guaranteed.

An acquisition is a specific example of a cooperative agreement between firms. Starting with Gans and Stern (2000), a long line of work has explored how ex post cooperative agreements between firms affect the ex ante incentives to innovate. Our contribution is to apply these ideas to the novelty of the innovation that is undertaken. This difference is most stark in the extreme case in which the incumbent acquires the entrant with the express intention of shutting it down, what Cunningham et al. (2020) have colorfully labelled “killer acquisitions.” The logic for killer acquisitions in our model is distinct from that of Cunningham et al. (2020) and, in contrast to their result, we find that killer acquisitions occur for a broader range of outcomes the more different is the entrant’s innovation from the incumbent’s product. The insight of Cunningham et al. (2020) is that development costs are more duplicative when technologies are more similar. We complement their explanation by showing that market competition effects alone are sufficient to justify a killer acquisition, and that this is more easily satisfied the more distant are the technologies.

Killer acquisitions occur when the entrepreneur’s innovation fails to meet expectations but only moderately so. When this occurs, the innovation delivers what we refer to as a “nuisance product.” It is a nuisance in that its lower quality adds no market-spanning advantage to the incumbent and, if the incumbent controlled the technology, it would shelve it and shut down the firm. Nevertheless, when controlled by the independent entrepreneur, the innovation is sufficiently strong to coexist in the market with the incumbent, engaging in competition and damaging the incumbent’s profit. This combination of factors is more easily satisfied the softer is price competition. Thus, the range of outcomes in which the entrant is a nuisance—and the incumbent desires a killer acquisition—the more distant is its innovation in the technology space.

A core part of our contribution is that we overlay technological differences on top of consumer preferences. In this way we combine the insights of Arrow and Hotelling into a single framework. We presume that this relationship is perfectly correlated, such that

greater technological distance moves in lockstep with more product differentiation. It is important to note that we do not impose this or any relationship on the outcomes of innovations, and nearby products in the technology space may very well produce radically different outcomes. Nevertheless, to differentiate products and soften competition, in our setting, firms must engage risk.<sup>5</sup>

### 1.1. Relationship to the Literature

A distinction between types of innovations has not gone unnoticed in the literature. Christensen (1997) famously articulated a typology of “sustaining” and “disruptive” innovations.<sup>6</sup> In Christensen’s telling, however, the distinction is binary and market disruption is possible only with disruptive innovations and not with sustaining ones. Our model shows that the distinction between sustaining and disruptive can be seen as one of degree rather than type, and that not only can an incumbent product be disrupted by an incremental innovation, it is, in fact, more likely to be so, even if is less dramatic than disruption that comes from bolder innovations.<sup>7</sup>

Several papers since Christensen have expanded the scope of an innovator’s choice set, although with different focus than our model. Bryan and Lemus (2017) allow firms to choose the direction of their innovation, where directions differ in the difficulty and value in completing a sequence of products. In Letina (2016), projects differ in their cost and, as in our model, firms choose projects in anticipation of the degree of future competition.<sup>8</sup> Our model of innovation differs in that we focus on novelty and, consequently, the risk of innovation. Our approach is akin to firms choosing how far along a given direction of innovation they wish to push rather than the choice of direction itself. In that sense, the paper closest to our is Cabral (2003), which allows firms to choose the variance of their innovation, although in that paper the choice is limited to be binary (high or low). The focus of the analysis is then how that choice changes over the dynamic course of market competition, and the model does not include horizontal differentiation and price competition that are central to our results.

Our disruption result is driven by the interaction of competition and the uncertainty of innovation. Disruption is not an end in itself, but rather a means toward profit maximization. Our model shows that when innovation is melded with spatial competition, the incentive to differentiate and soften competition works against disruption. This creates a wedge that leads to the more novel innovation having the lower probability of disruption. This mechanism is fundamentally different from the limited cognition explanation in Christensen (1997).<sup>9</sup>

Gans and Stern (2000) initiated the study of cooperative agreements among competitors and the impact these agreements have on ex ante incentives to innovate (see Gans et al. 2002 for empirical exploration of this relationship).<sup>10</sup> The set of cooperative tools is large and, in focusing only on acquisitions, we take just a first step in connecting the novelty of innovation to broader firm strategy. Gans (2017) characterizes the differences between acquisitions and other cooperative tools. Exploring how these differences impact the incentives to innovate that we identify here is an interesting open question.

Following the Gans and Stern (2000) framework, a recent set of papers explores the impact of cooperative agreements, and acquisitions in particular, on innovation and market competition. We complement this literature by adding the novelty dimension to this question and show how the novelty of innovation can be suppressed at the same time as the incentive to innovate increases. Bryan and Hovenkamp (2020a) is the closest to our paper, showing how lax antitrust policy can distort the direction of innovation in a way that is analogous to the effect we identify for the novelty of innovation.<sup>11</sup> We identify a connection between Arrow and Hotelling and focus on the relationship between innovation and entry. Bryan and Hovenkamp (2020a), in contrast, deftly separate the effects of entry from innovation by supposing that the innovator licenses its technology to one or both existing incumbent firms, thereby providing a clearer picture of how innovation depends on competition.

## 2. A Model of Innovation Novelty

We study competition between two firms, an incumbent and an entrant. The firms locate in the technology space, which is given by the interval  $[-\frac{1}{2}, \frac{1}{2}]$ . Consumers have heterogeneous preferences over the products these technologies produce, with their ideal points distributed uniformly over the space.

The incumbent firm makes product 0, which is located at the center of the market, denoted  $l_0 = 0$ , with known quality  $v_0$ . The entrant makes product 1 and enters the market by choosing a technology with which to produce this product. Without loss of generality, we set this to be  $l_1 \in [0, \frac{1}{2}]$ . Any product other than 0 is new and, thus, has unknown quality. We suppose that this uncertainty increases in the novelty of the innovation, which is measured by its distance from the technology underlying product 0. Specifically, after the entrant chooses  $l_1$ ,  $v_1$  is drawn from a normal distribution with mean  $v_0 + \mu l_1$  and variance  $\sigma^2 l_1$ . The variance parameter  $\sigma^2 > 0$  captures the notion that there is more uncertainty about the quality of a new product the further it is located from an existing one. The parameter  $\mu \geq 0$ , in turn, captures the extent

to which differentiation increases the new product's expected quality.<sup>12</sup> We denote the quality differential between the products as:

$$\Delta \equiv v_1 - v_0.$$

For simplicity, we set the marginal cost of production to zero for both firms.<sup>13</sup>

The entrant incurs development costs  $c(l_1) \geq 0$  for its new product. These costs are larger the further the new product is from the existing one, reflecting the fact that it takes time and resources to research into the unknown. In particular,  $c(0) = c'(0) = 0$  and  $c'(l_1) \geq 0$  for all  $l_1 \in [0, \frac{1}{2}]$ . Furthermore, we assume  $c(l_1)$  is sufficiently convex for the entrant's location problem to be well-behaved, and we make this precise in the appendix.

Each consumer,  $s$ , buys at most one unit of either product 0 or 1. The utility from product  $j \in \{0, 1\}$  at price  $p_j$ , decreases quadratically in the distance of that product from the consumer's ideal, as in the classic Hotelling (1929) model of spatial competition. Specifically,

$$u_{sj} = v_j - t(s - l_j)^2 - p_j,$$

where  $t \geq \mu \geq 0$  represents the disutility of technology distance.<sup>14</sup> We require  $t \geq \mu$  as otherwise there is no trade-off between technology and consumers. The reservation utility of buying neither product is zero. To ensure that all consumers buy one of the products, as is standard in Hotelling models, we assume that the quality of the incumbent's product satisfies<sup>15</sup>:

$$v_0 \geq \frac{5}{4}t. \quad (1)$$

We are interested in how ownership of the entrant affects innovation and market outcomes. To this end, we compare three cases: (i) the entrant is independent, (ii) the incumbent owns the entrant, and (iii) the entrant is independent initially but can be acquired by the incumbent after its product has been developed and before prices are set. In the last case, the acquisition price is determined by Nash bargaining, where the entrant's bargaining power is given by  $\alpha \in [0, 1]$  and the incumbent's by  $(1 - \alpha)$ .

At the beginning of the game, the entrant decides on the location of its product, after which the product's quality is realized. Depending on the case we examine, the incumbent may then be able to acquire the entrant. Next, the firms simultaneously set a single price for all consumers  $p_0 \in [0, \infty)$  and  $p_1 \in [0, \infty)$  and do so noncooperatively if the entrant is independent. Finally, consumers make their purchase decisions, profits are realized, and the game ends. We use the notation  $\pi_j^M$  and  $\pi_j^C$  to denote the gross profit—profit excluding development costs—for product  $j \in \{0, 1\}$

when the market is monopolistic and competitive, respectively.

We characterize the unique subgame perfect equilibrium for all permutations of the model. The derivation proceeds by backward induction, and we follow this logic in presenting the results, beginning with price setting by the incumbent and the entrant once qualities are determined.

### 3. Prices, Market Shares, and Profits

As a benchmark, consider the incumbent's problem if the new product did not exist. In this case, the incumbent would charge the highest price at which all consumers are willing to buy the existing product,

$$\bar{p}_0 = v_0 - \frac{1}{4}t.$$

A higher price would cause the incumbent to lose marginal consumers on the flank, and the condition in (1), along with the uniform distribution of consumers, implies this is not profitable. Charging a lower price would not win any new customers and would simply turn profits into consumer surplus. The price  $\bar{p}_0$  generates profit  $\bar{\pi}_0$ , and since there is a unit mass of consumers and zero marginal cost of production, we have  $\bar{\pi}_0 = \bar{p}_0$ . In what follows, we refer to  $\bar{p}_0$  and  $\bar{\pi}_0$  as the status quo price and profits.

Suppose now the new product does exist and is either owned by the independent entrant or the incumbent. Fixing the entrant's location at  $l_1 > 0$ , Figure 1 illustrates equilibrium prices and the implied market share for the new product for the two cases (see Lemmas A.1 and A.2 in the appendix for the corresponding expressions).

Begin with the case in which the incumbent owns the new product. As the figure indicates, the incumbent continues to charge the status quo price  $\bar{p}_0$  for the existing product as this appeals to the consumers on the left flank and ensures they all make a purchase.

As Arrow pointed out, sales of the new product necessarily come at the expense of the existing product. To be profitable, therefore, the price charged for the new product must be at least as high as for the existing product. As consumers are closer to the entrant technologically, the entrant can capture market share from these consumers even if its quality is lower. The quality differential cannot be too large, however, for this to be the case. Specifically, the new product fails to capture any market share if the quality differential,  $\Delta \equiv v_1 - v_0$ , satisfies

$$\Delta \leq -tl_1(1 - l_1). \quad (2)$$

Similarly, the existing product will not be able to hold on to any market share if the quality of the new product is sufficiently high that

$$\Delta \geq tl_1(3 + l_1). \quad (3)$$

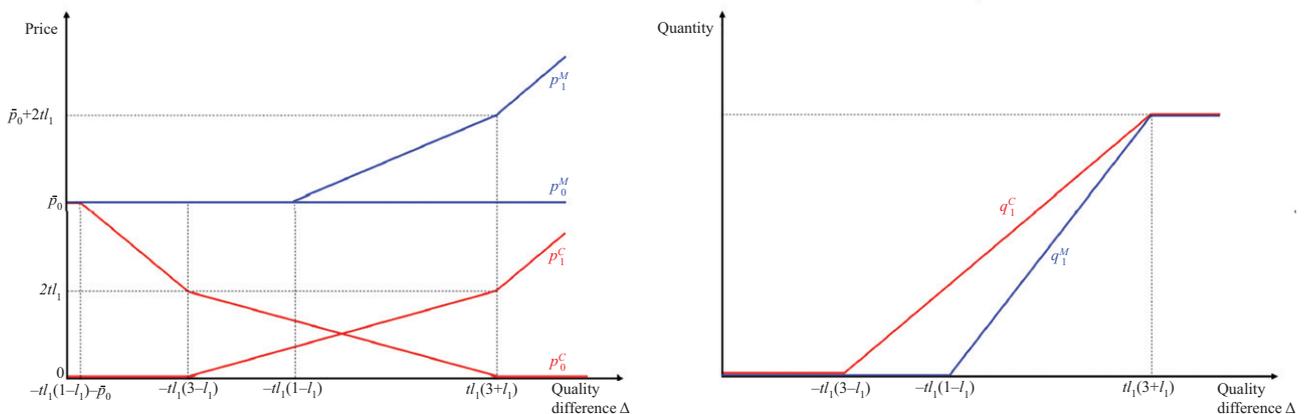
In between these two thresholds, the incumbent sells both products and it sells more of the new product, and at a higher price, the larger the quality differential. Lemma 1 describes the profit to the incumbent from this pricing strategy.

**Lemma 1.** *If the incumbent owns the entrant, its gross profit is*

$$\pi_0^M + \pi_1^M = \begin{cases} \bar{\pi}_0 + \Delta - tl_1(1 + l_1) & \text{if } \Delta \geq tl_1(3 + l_1) \\ \bar{\pi}_0 + \frac{1}{8tl_1}(\Delta + tl_1(1 - l_1))^2 & \text{if } -tl_1(1 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ \bar{\pi}_0 & \text{if } \Delta \leq -tl_1(1 - l_1). \end{cases}$$

When the entrant is independent, the two firms compete over price within the market. As a result, prices fall and the new product captures a larger share of the market. The market share and price of the new product are again increasing in the quality differential  $\Delta$  and the new product still takes over the entire market if the differential satisfies (3). The lower threshold below which the new product fails to capture any

**Figure 1.** (Color online) Equilibrium Prices and Market Shares for Fixed  $l_1 > 0$



market share, though, is lower. Specifically, under competition the new product fails to capture any market share if

$$\Delta \leq -tl_1(3 - l_1), \quad (4)$$

as illustrated in Figure 1.

If the quality differential is between the two lower thresholds, that is, if  $-tl_1(3 - l_1) \leq \Delta \leq -tl_1(1 - l_1)$ , the new entry is what we call a nuisance product. Its quality is high enough to steal market share from the incumbent when the firms are competing, but not high enough to attract any consumers at the high price the incumbent would want to sell it at in the absence of any competitive pressure.

When (4) holds, the entrant fails to capture market share even if it gives the new product away for free. It nevertheless can still apply pricing pressure on the incumbent. In the range

$$-tl_1(1 - l_1) - \bar{p}_0 < \Delta \leq -tl_1(3 - l_1), \quad (5)$$

the new product at a zero price constrains what the incumbent can charge for the existing product, and the incumbent is forced to lower the price below the status quo. In this case, the new product contests the market for the existing one despite not gaining any market share. It is only if  $\Delta \leq -tl_1(1 - l_1) - \bar{p}_0$  and below the lower threshold that the new product does not contest the market and the incumbent obtains the status quo profit.

Lemma 2 describes the profit from this competition for the two firms.

**Lemma 2.** *If the entrant is independent, the incumbent's gross profit is*

$$\pi_0^C = \begin{cases} 0 & \text{if } \Delta \geq tl_1(3 + l_1) \\ \frac{1}{18tl_1} (\Delta - tl_1(3 + l_1))^2 & \text{if } -tl_1(3 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ -\Delta - tl_1(1 - l_1) & \text{if } -tl_1(1 - l_1) - \bar{p}_0 \leq \Delta \leq -tl_1(3 - l_1) \\ \bar{\pi}_0 & \text{if } \Delta \leq -tl_1(1 - l_1) - \bar{p}_0 \end{cases}$$

and the entrant's gross profit is

$$\pi_1^C = \begin{cases} \Delta - tl_1(1 + l_1) & \text{if } \Delta \geq tl_1(3 + l_1) \\ \frac{1}{18tl_1} (\Delta + tl_1(3 - l_1))^2 & \text{if } -tl_1(3 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ 0 & \text{if } \Delta \leq -tl_1(3 - l_1). \end{cases}$$

Notice that, in both lemmas, profits are convex in the quality differential  $\Delta$  when the market share of each product is strictly positive. This reflects our previous observation that, for this case, both the price and market share of the new good are increasing in  $\Delta$  and those of the existing product are decreasing. As a result, the firms are risk loving with respect to  $\Delta$  (despite being risk neutral over profit itself).

This property delivers a distinct force toward innovation. The entrant has an incentive to innovate boldly

as doing so increases risk. The upside is a breakthrough innovation that not only creates a lot of value but allows the entrant to capture that value as it dominates the incumbent and competition is weak. The downside is a failed innovation, although the direct loss is bounded below by the development costs. Moreover, the missed profit, relative to a safer innovation, is relatively small given the intensity of competition that would otherwise emerge. This incentive to innovate is suppressed in models of step-by-step innovation, such as in the quality ladder literature.

## 4. Innovation

At the beginning of the game, the entrant chooses the location of the new product. To maximize profit, the incumbent maximizes the expected gain from innovation, where the gain is  $G^M = \pi_0^M + \pi_1^M - \bar{\pi}_0$ , minus the cost of product development,  $c(l_1)$ . Thus, when the incumbent owns the entrant, the optimal location  $l_1^M$  maximizes

$$E[G^M] - c(l_1). \quad (6)$$

If, instead, the entrant is independent, the gain from innovation is simply  $G^C = \pi_1^C$ , and the optimal location is the  $l_1^C$  that maximizes

$$E[G^C] - c(l_1). \quad (7)$$

The assumption that  $c(l_1)$  is sufficiently convex for the objective functions to be strictly concave ensures that the optimal locations are unique. We can now establish the spatial Arrow effect.

**Proposition 1.** *The new product is more differentiated from the existing one if the entrant is independent than if it is owned by the incumbent, that is,*

$$l_1^C > l_1^M.$$

To see why  $l_1^C > l_1^M$ , it is helpful to break the firm's problem down into competition and innovation effects and to separate these from the development costs. We begin with competition.

Fix the location of  $l_1$  and set the quality differential  $\Delta = v_1 - v_0$  to a specific value so that the outcome of innovation has already been realized. Taking the difference between the gains from innovation for each type of entrant and rearranging, we obtain the following:

$$G^C - G^M = \underbrace{(\bar{\pi}_0 - \pi_0^C)}_{\text{replacement}} - \underbrace{[(\pi_0^M + \pi_1^M) - (\pi_0^C + \pi_1^C)]}_{\text{gains from coordinating prices}}. \quad (8)$$

Decomposing in this way allows us to see the dueling effects on innovation. The independent entrant misses out on the benefit from coordinating prices, whereas it avoids the loss of profit on the existing product—the

cost of replacement—that is borne by the incumbent. Overall, the replacement effect dominates so that  $G^C - G^M \geq 0$ . In line with Arrow’s classic argument, the independent entrant gains more from innovation. This ordering is depicted in Figure 2 for two locations of the entrant.

What drives the result in Proposition 1, though, is not the difference in the levels of the gains from innovation but in the margins. To show that this holds, we differentiate (8) in location  $l_1$  and establish that it is weakly positive for all  $l_1 \in [0, \frac{1}{2}]$ :

$$\begin{aligned} \frac{d}{dl_1}(G^C - G^M) &= \frac{d}{dl_1}(\bar{\pi}_0 - \pi_0^C) \\ &\quad - \frac{d}{dl_1}[(\pi_0^M + \pi_1^M) - (\pi_0^C + \pi_1^C)] \\ &\geq 0. \end{aligned} \quad (9)$$

Both terms in the derivative are in general negative.<sup>16</sup> The replacement cost decreases as the new product is more differentiated, capturing Arrow’s effect. On its own, this effect implies a larger marginal gain from innovation for the incumbent entrant. The benefit of coordinating prices, however, also decreases in differentiation, reflecting Hotelling’s classic insight. This force implies a larger marginal gain from innovation for the independent entrant. In the proof of Proposition 1, we show that the latter effect strictly dominates the former for any parameter values, except when only one product is sold, in which case they are the same. The independent entrant’s desire to soften competition, therefore, outweighs the incumbent’s desire to protect the existing product. This relative change is evident across the two panels of Figure 2.

Allowing for uncertainty does not affect this ordering. The previous argument is for any fixed quality of the new product, whereas an increase in the novelty of innovation also increases both the variance in outcomes and the mean through the drift term,  $\mu$ . Nevertheless, we show that the relationship in (9) continues to hold once we take uncertainty into account. That is, we have that

$$\frac{d}{dl_1}E(G^C - G^M) \geq 0 \quad (10)$$

for all  $l_1 \in [0, \frac{1}{2}]$  and the marginal incentive for the independent entrant is strictly higher than for the incumbent.

The final step in the argument is to incorporate development costs,  $c(l)$ . As  $l_1$  increases and the novelty of the innovation increases, the firms trade off increasing gains from innovation against increasing cost of development. As both types of entrant face the same development cost function, (10) implies that the independent entrant is more willing to pay that cost and innovate boldly. The ordering in Proposition 1 follows.

#### 4.1. Disruption

The uncertainty of innovation matters also for disruption. A natural intuition is that the independent entrant is more like to disrupt the incumbent—to drive it from the market—because of the greater risk from innovation. The entrepreneur has a greater chance of failure, but also a greater chance of achieving a breakthrough, and it is breakthrough innovations that disrupt the market.

This intuition is true all else equal, but in markets where innovation and competition are both spatial, all else is not equal. The independent entrant innovates more boldly and, therefore, competes less intensely with the incumbent product. This differentiation makes disruption less likely for the entrepreneur. The impact on disruption depends, therefore, on the balance between these two forces.

The effect of uncertainty can be seen from the normal distribution, as depicted in Figure 3. Let  $f(\Delta)$  and  $F(\Delta)$  denote the probability density function (PDF) and cumulative density function (CDF) of a normal distribution with mean  $\mu l_1$  and variance  $\sigma^2 l_1$ . Then, for any fixed threshold  $\delta > 0$ ,

$$\Pr(\Delta \geq \delta) = 1 - F(\delta)$$

and

$$\frac{d\Pr(\Delta \geq \delta)}{dl_1} = \frac{\delta + \mu l_1}{2\sigma l_1 \sqrt{l_1}} f(\delta) > 0.$$

This captures the intuition that bolder innovations having more chance of a breakthrough (and, conversely, more chance of being a flop).

However, as discussed, recall from (3) that the existing product has zero market share and is wiped out if  $\Delta \geq tl_1(3 + l_1)$ , regardless of whether the entrant is independent or not. Critically, this requirement is quadratic in  $l_1$ , as depicted in Figure 3.

The probability of disruption, then, is the probability that the quality of the entrant exceeds this increasing threshold. As the novelty of the innovation increases, this becomes a race between the threshold and increasing variance. Formally, the probability of disruption is

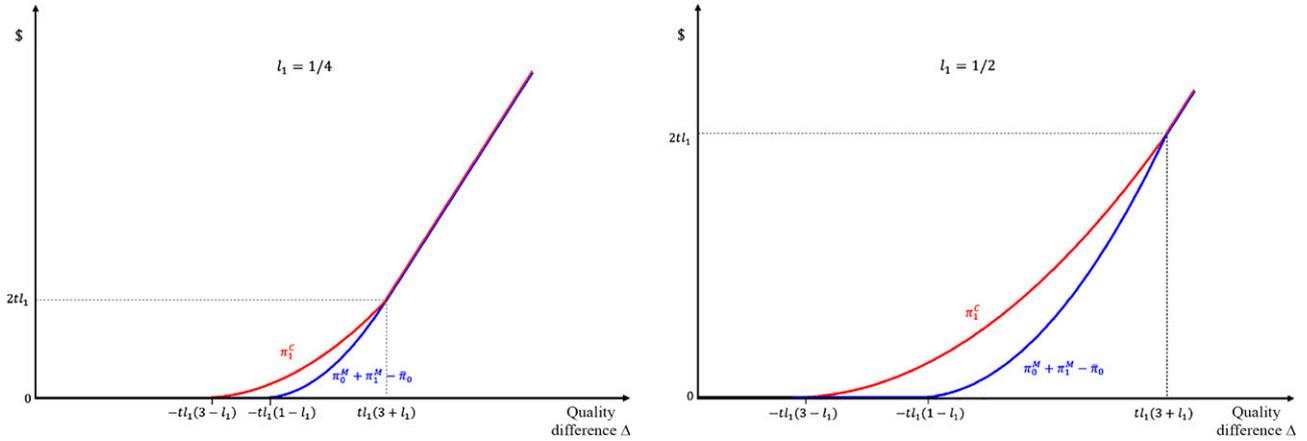
$$\Pr(q_0^j = 0) = 1 - F\left(tl_1(3 + l_1)\right) \text{ for } j = C, M.$$

Differentiating shows that the race is won by the threshold as the probability of disruption is decreasing in  $l_1 > 0$ :

$$\frac{d\Pr(q_0^j = 0)}{dl_1} = -\frac{\mu + 3tl_1(1 + l_1)}{2\sigma l_1 \sqrt{l_1}} f\left(tl_1(3 + l_1)\right) < 0.$$

From Proposition 1, the following result holds.

**Corollary 1.** *The existing product’s market share is more likely to be zero when the entrant is incumbent owned than when it is independent. Conversely, the entrant’s market*

**Figure 2.** (Color online) Gains from Innovation

Note. In the graph on the left,  $v_0 = \frac{3}{2}$ ,  $t = 1$ , and  $l_1 = \frac{1}{4}$ ; on the right,  $l_1 = \frac{1}{2}$ .

share is more likely to be zero when the entrant is incumbent owned than when it is independent.

The converse implies that the independent entrant is less likely to completely fail—to disappear from the market—than is the incumbent-owned entrant. It follows from an identical procedure to that for disruption and, in fact, holds with even greater force. The entrepreneur has a higher chance of a bad outcome, but in a reflection of that for disruption, the threshold for complete failure is decreasing and concave, and this effect dominates the increased variance. On top of this, the threshold for the incumbent-owned entrant to achieve a positive market share is higher than for an independent entrepreneur (recall Figure 1), whereas both types of entrant faced the same threshold on the positive side for disruption.

Corollary 1 holds for any value of the drift term,  $\mu$  (that satisfies our requirement  $\mu \leq t$ ). As  $\mu$  varies, the threshold for disruption and failure remain the same. Thus, the probability of disruption and failure adjust accordingly, increasing and decreasing in  $\mu$ , respectively. For very promising technological directions, the probability of disruption is high, whereas in most markets a smaller  $\mu$  is more descriptive and innovation is marked more by failure than disruption.

That incumbent-owned innovation is more likely to fail may shed light on why incumbent firms are so often thought to be bad at innovation and to engage in so many failures. As the model illustrates, incumbents rationally innovate incrementally, and this choice sets a high bar for success, a higher bar than for independent entrants who have more competitive freedom precisely because they innovate more boldly.

## 5. Acquisitions and Antitrust

Suppose now the entrant owns the new product but can be acquired by the incumbent. Since the firms must be jointly better off coordinating prices, and there are no diseconomies of scale, the firms' joint gains from merging are always positive, that is,

$$\pi_0^M + \pi_1^M - (\pi_0^C + \pi_1^C) \geq 0.$$

Moreover, since there are no informational asymmetries or other bargaining frictions, the parties always agree on a merger.

The firms Nash bargain over the gains, where the independent entrant's bargaining power is given by  $\alpha \in [0, 1]$  and the incumbent's by  $1 - \alpha$ . If the incumbent and the entrant do not agree on a merger, they compete for customers and realize profits  $\pi_0^C$  and  $\pi_1^C$ . Given these outside options, and the quality of the new product, the incumbent's expected profits are then

$$\pi_0^C + (1 - \alpha)(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C)$$

and the entrant's are

$$\pi_1^C + \alpha(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C).$$

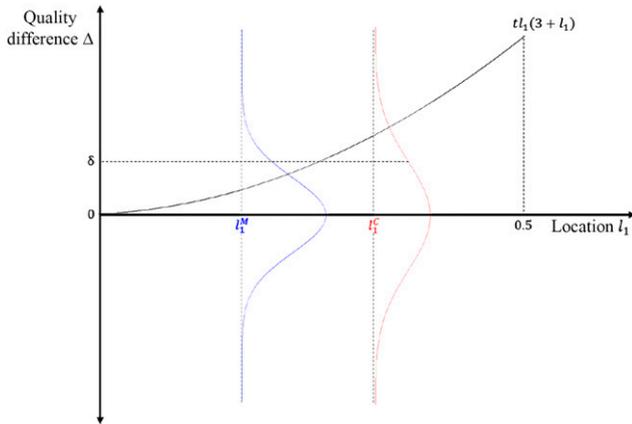
Even though the parties always agree to merge, the motive and implications of the merger depend on the location and quality of the new product, as is summarized in Figure 4. The key thresholds in this calculus include the cut points in market share expressed earlier in (2), (3), (4), and (5), and reflected in Figure 1.

Fixing  $l_1$ , we begin with the lowest realizations of  $\Delta$ . For

$$\Delta \leq -t_1(1 - l_1) - \bar{p}_0,$$

the quality of the entrant's innovation is sufficiently low that it not only fails to gain any market share, but

**Figure 3.** (Color online) Disruption Probability,  $\mu = 0$



fails to restrain the incumbent’s pricing power in any way. This is the lowest threshold in Figure 4. This occurs when no consumer is willing to buy the new product even if it is given away for free and the existing product is sold at the status quo price  $\bar{p}_0$ . The gains to a merger in this case is zero. The firms are not hurt by merging, but they also don’t benefit from it, and the entrant receives a price of zero upon merging.

For a quality differential that is somewhat higher, in particular, if (5) holds:

$$-tl_1(1 - l_1) - \bar{p}_0 < \Delta \leq -tl_1(3 - l_1),$$

the entrant contests the market despite obtaining zero market share, thereby limiting the pricing power and the profit of the incumbent product. Acquiring the entrant doesn’t increase the incumbent’s market share but it does allow it to remove this impediment to high monopoly prices. The merger is valuable, therefore, and the entrant receives a positive price despite it having no profitable future as an independent company.

Suppose next that the quality differential is good enough to capture market share in a competitive industry but not good enough to attract consumers at the high price the monopolist would sell it at. That is, suppose the quality differential is between the thresholds given by (2) and (4),

$$-tl_1(3 - l_1) < \Delta \leq -tl_1(1 - l_1).$$

As we saw earlier, the new entrant is then a nuisance product. (This can be seen in the right-side panel in Figure 1.) In this case, the incumbent acquires the entrant to shut it down. In the terminology of Cunningham et al. (2020), the incumbent engages in a killer acquisition. This possibility emerges if and only if the innovation fails to meet expectations, but only slightly so. The range when it is possible, as indicated in

Figure 4 and explained in the introduction, expands with the novelty of the innovation.

Acquiring the new product to kill it no longer makes sense for the incumbent if the quality differential is larger. In particular, if

$$-tl_1(1 - l_1) < \Delta < tl_1(3 + l_1),$$

the new product is sufficiently valuable that some consumers are willing to buy it at more than the status quo price (the differential  $\Delta$  is between the thresholds in (2) and (3)). This is the standard case for a market power merger as it allows the firm to charge a higher price for both products without eliminating either from the market. The new product is valuable even if it is the same quality as it enables the incumbent to serve more of the market with products tailored to separate segments.

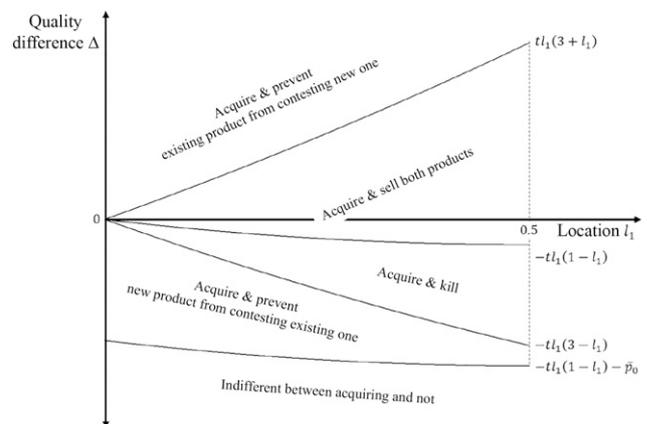
The final case is when the quality of the entrant’s innovation is so high that it dominates the market and the incumbent product is disrupted. This is exactly (3):

$$\Delta \geq tl_1(3 + l_1).$$

Although the incumbent is disrupted, the merger is valuable as the market is still contested, although this time it is the market for the new product that is contested rather than the market for the existing product. The incumbent receives a positive price regardless of the quality of the new product as its product remains valuable to consumers for a sufficiently low but still positive price. This holds for arbitrarily high quality of the new product and there is no symmetric case to when the entrant’s low quality renders it irrelevant.<sup>17</sup>

With the acquisition strategy of the firms in hand, we can now, by backward induction, determine the impact mergers have on the incentive to innovate, in particular on the novelty of the innovation a new entrant undertakes.

**Figure 4.** Acquisition Strategy of the Incumbent Firm



As we did in Section 4, the entrepreneur chooses a new technology to maximize profitability. In this case, the entrepreneur does so with postinnovation and postmerger outcomes in mind. The entrepreneur's optimal choice is now given by

$$l_1^A(\alpha) = \max_{l_1 \in [0, \frac{1}{2}]} E[\pi_1^C + \alpha(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C)] - c(l_1),$$

where the superscript  $A$  stands for acquired. We can now establish the result.

**Proposition 2.** *An independent entrant that can be acquired has an optimal location  $l_1^A(\alpha)$  that is decreasing in its bargaining power and satisfies  $l_1^A(0) = l_1^C$  and  $l_1^A(1) \in (0, l_1^M)$ .*

This says that the spatial Arrow effect is reversed if the entrant has sufficient bargaining power. The independent entrant innovates more incrementally and with less novelty than would the incumbent firm if it controlled the entrant from the start. This effect increases in the entrant's bargaining power.

For the characterization of the choice  $l_1^A$ , notice that we can rewrite the entrant's gains from innovation as

$$\begin{aligned} & E[\pi_1^C + \alpha(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C)] \\ &= E[(1 - \alpha)\pi_1^C + \alpha(\pi_0^M + \pi_1^M) - \alpha\pi_0^C]. \end{aligned}$$

If the entrant has no bargaining power,  $\alpha = 0$ , the entrant expects to make expected gross profits  $E[\pi_1^C]$ , and thus chooses the same location  $l_1^A(0) = l_1^C$ , as when it cannot be acquired. If, instead, the entrant has all the bargaining power, its expected gross profits are  $E[\pi_0^M + \pi_1^M - \pi_0^C]$ , which is less than the expected gross profits,  $E[\pi_0^M + \pi_1^M]$ , the incumbent expects to make when it owns the entrant from the start. Now notice that  $E[\pi_0^C]$  is increasing in  $l_1$ : the more the new product is differentiated from the existing one, the higher the incumbent's expected gross profits if it ends up competing with the independent entrant. It then follows that an entrant who knows it will be acquired by the incumbent chooses a location to the left of the one chosen by the incumbent-owned entrant,  $l_1^A(1) < l_1^M$ . The reason is that locating closer to the existing product lowers the incumbent's outside option, the profits it can expect to make if it does not acquire the entrant. This lower outside option, in turn, forces it to pay a higher price for the entrant.

### 5.1. Consumer Surplus

The connection between the novelty of an innovation and product differentiation affects consumers as well. The greater novelty of the independent entrant improves consumer welfare through the promise of a breakthrough innovation, whereas the cost of failure is bounded by the existence of the incumbent product. Greater novelty also imposes a cost on consumers.

Differentiation means softer price competition, and by innovating toward the fringes of the consumer distribution, any success fits less well the preferences of the average consumer. Nevertheless, we show that the impact of competition itself is strong enough that consumers are better off with an independent entrant than with an acquired entrant when the Arrow effect is reversed.

**Proposition 3.** *Expected consumer surplus is larger when the entrant stays independent than when it is acquired by the incumbent.*

This implies immediately that consumer surplus is higher with an independent entrant than one controlled by the incumbent. It is worth noting that Proposition 3 holds despite innovation and competition bringing no new consumers into the market. The covered-market assumption implies that consumer surplus is determined only by how well existing customers are served, not because the lower prices or differentiated product alleviates the market-limiting effects of monopoly pricing.<sup>18</sup>

### 5.2. Disruption

We saw earlier that an independent entrant (that cannot be acquired) disrupts the existing product less frequently than does the incumbent-owned entrant. The same logic holds here, although now, when the Arrow effect is reversed, it is the entrant who has a higher chance of disruption. However, we should not laud such entrepreneurs. For despite disrupting the incumbent, they are able to do so only because they innovate so tepidly, varying the existing product only incrementally. It is precisely because they locate so close to the existing product that only a small quality premium is needed to disrupt. The rent-seeking behavior of such entrepreneurs may be more likely to lead to market disruption—and it certainly increases their own pay-off—but it lowers overall innovation and market efficiency, and we are all the lesser for it.

## 6. Conclusion

The question of antitrust policy in innovative industries has recently received considerable attention, both in academic research and in the popular domain. The issues at the heart of this debate match the object of our analysis. Is a market more innovative when controlled by an incumbent with market power? Or is innovation better served by creating space for entrepreneurial upstarts and market competition? These questions of antitrust reach a climax when policymakers must decide whether to allow the incumbent to take over the upstart. Should Facebook be allowed to buy Instagram, Amazon buy Zappos, or AT&T buy T-Mobile? In expanding analysis to include the novelty

of innovation, our model provides an additional dimension by which policymakers can understand more deeply the market and the incentives that antitrust policy provides.

In expanding the domain of analysis for innovation and antitrust policy, we set aside other important dimensions, and there is much that our model leaves out. One prominent feature we leave out is market competition dynamics. Many researchers emphasize the dynamics of disruption that come from new innovations (Christensen 1997, Adner and Zemsky 2005). In modeling only a single period of market competition, our model cannot speak to this feature, and extending our model of innovation novelty in this direction offers substantial promise. Nevertheless, the static analysis provides more than just a starting point as it illuminates what role dynamics actually play in disruption.<sup>19</sup> We show that disruption can emerge in a single period and that it can come from the ends of the market, as Christensen suggests it typically does, even if it is immediate in the model whereas it takes some time in practice.

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### Appendix

**Lemma A.1.** *If the incumbent owns the entrant, it charges  $p_0^M = \bar{p}_0$  for the existing product and*

$$p_1^M = \begin{cases} \bar{p}_0 + \Delta - tl_1(1 + l_1) & \text{if } \Delta \geq tl_1(3 + l_1) \\ \bar{p}_0 + \frac{1}{2}(\Delta + tl_1(1 - l_1)) & \text{if } -tl_1(1 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ \bar{p}_0 & \text{if } \Delta \leq -tl_1(1 - l_1) \end{cases}$$

for the new one. The market share of the new product is given by

$$q_1^M = \begin{cases} 1 & \text{if } \Delta \geq tl_1(3 + l_1) \\ \frac{1}{2} + \frac{1}{4tl_1}(\Delta - tl_1(1 + l_1)) & \text{if } -tl_1(1 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ 0 & \text{if } \Delta \leq -tl_1(1 - l_1) \end{cases}$$

**Proof.** The consumer indifferent between consuming products 0 and 1 is located at

$$x = \frac{1}{2}l_1 - \frac{1}{2tl_1}((v_1 - p_1) - (v_0 - p_0)). \quad (\text{A.1})$$

The firm's profit maximization problem is then given by

$$\max_{p_0, p_1} p_0 \left( x + \frac{1}{2} \right) + p_1 \left( \frac{1}{2} - x \right),$$

which, after substituting  $x$  gives,  $p_0 = \bar{p}_0$ ,

$$p_1 = \left( \frac{1}{4} \left( 2(v_1 + v_0) - t + 2tl_1(1 - l_1) \right) \right),$$

and

$$x = \frac{1}{4tl_1} (v_0 - v_1 + tl_1(l_1 + 1)).$$

This results in three different regimes: (i)  $x \geq 1/2$  when  $\Delta \geq tl_1(3 + l_1)$ , where the entrant captures the entire market, (ii)  $x \in (-1/2, 1/2)$  when  $\Delta \in (-tl_1(1 - l_1), tl_1(3 + l_1))$ , where both incumbent and entrant have positive market shares, and (iii)  $x \leq -1/2$  when  $\Delta \leq -tl_1(1 - l_1)$ , where the incumbent captures the entire market.  $\square$

**Lemma A.2.** *If the entrant is independent, the incumbent charges*

$$p_0^C = \begin{cases} 0 & \text{if } \Delta \geq tl_1(3 + l_1) \\ \frac{1}{3}(-\Delta + tl_1(3 + l_1)) & \text{if } -tl_1(3 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ -\Delta - tl_1(1 - l_1) & \text{if } \frac{1}{4}t - tl_1(1 - l_1) - v_0 \leq \Delta \leq -tl_1(3 - l_1) \\ \bar{p}_0 & \text{if } \Delta \leq -tl_1(1 - l_1) - \bar{p}_0 \end{cases}$$

and the entrant charges

$$p_1^C = \begin{cases} \Delta - tl_1(1 + l_1) & \text{if } \Delta \geq tl_1(3 + l_1) \\ \frac{1}{3}(\Delta + tl_1(3 - l_1)) & \text{if } -tl_1(3 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ 0 & \text{if } \Delta \leq -tl_1(3 - l_1) \end{cases}$$

The market share of the new product is given by

$$q_1^C = \begin{cases} 1 & \text{if } \Delta \geq tl_1(3 + l_1) \\ \frac{1}{2} + \frac{1}{6tl_1}(\Delta - tl_1^2) & \text{if } -tl_1(3 - l_1) \leq \Delta \leq tl_1(3 + l_1) \\ 0 & \text{if } \Delta \leq -tl_1(3 - l_1) \end{cases}$$

**Proof.** Given the location (A.1) of the consumer who is indifferent between the two products, the incumbent's problem is given by

$$\max_{p_0} p_0 \left( \frac{1}{2} + \frac{1}{2tl_1} (v_0 - v_1 + p_1 - p_0 + l_1^2 t) \right),$$

and the entrant's problem is given by

$$\max_{p_1} p_1 \left( \frac{1}{2} - \frac{1}{2tl_1} (v_0 - v_1 - p_0 + p_1 + l_1^2 t) \right).$$

This results in

$$p_0 = \frac{1}{3} (v_0 - v_1 + tl_1(3 + l_1)),$$

$$p_1 = \frac{1}{3} (v_1 - v_0 + tl_1(3 - l_1)),$$

and

$$x = \frac{1}{2} - \frac{1}{6tl_1} (v_1 - v_0 + tl_1(3 - l_1)).$$

We again have three regions: (i)  $x \geq 1/2$  when  $\Delta \geq tl_1(3 + l_1)$ , where the entrant captures the entire market, (ii)  $x \in (-1/2, 1/2)$  when  $\Delta \in (-tl_1(3 - l_1), tl_1(3 + l_1))$ , where both incumbent and entrant have positive market shares, and (iii)  $x \leq -1/2$  when  $\Delta \leq -tl_1(3 - l_1)$ , where the incumbent captures the entire market. Moreover, in the incumbent-only region, the incumbent is constrained by

the consumer at 1/2 if the consumer prefers the entrant’s product at  $p_1 = 0$  over the incumbent’s product, which holds for the range  $-tl_1(1 - l_1) - \bar{p}_0 \leq \Delta \leq -tl_1(3 - l_1)$ .  $\square$

**Proof of Lemmas 1 and 2.** Both follow from Lemmas A.1 and A.2.  $\square$

**Proof of Proposition 1.** We use Lemmas 1, 2, A.1, and A.2 to calculate the difference in expected gain from innovation, which results in

$$\begin{aligned} & E(\pi_1^C) - E(\pi_0^M + \pi_1^M - \bar{\pi}_0) \\ &= \int_{v_0 - l_1 t(3 - l_1)}^{(v_0 - l_1 t(1 - l_1))} \underbrace{\frac{1}{3}(\Delta + tl_1(3 - l_1))}_{p_1^C} \underbrace{\frac{1}{6tl_1}(\Delta + tl_1(3 - l_1))}_{q_1^C} dF(\Delta) \\ &+ \int_{(v_0 - l_1 t(1 - l_1))}^{(v_0 + l_1 t(l_1 + 3))} \underbrace{\frac{1}{3}(\Delta + tl_1(3 - l_1))}_{p_1^C} \underbrace{\frac{1}{6tl_1}(\Delta + tl_1(3 - l_1))}_{q_1^C} \\ &- \underbrace{\frac{1}{2}(\Delta + tl_1(1 - l_1))}_{p_1^M - p_0^M} \underbrace{\frac{1}{4tl_1}(\Delta + tl_1(1 - l_1))}_{q_1^M} dF(\Delta), \end{aligned}$$

where  $F(\Delta)$  is the CDF for  $N(\mu l_1, l_1 \sigma^2)$ .

Next, we use change of variables  $\Delta = \mu l_1 + \sqrt{l_1} \sigma z$  to get the integrals in terms of standard normal distributed variable  $z$ :

$$\begin{aligned} & E(\pi_1^C) - E(\pi_0^M + \pi_1^M - \bar{\pi}_0) \\ &= \int_{v_0 - l_1 t(3 - l_1)}^{(v_0 - l_1 t(1 - l_1))} \underbrace{\frac{\sigma \sqrt{l_1}}{3} \left( z - \frac{-tl_1(3 - l_1) - \mu l_1}{\sigma \sqrt{l_1}} \right)}_{p_1^C} \\ &\quad \underbrace{\left( \frac{\sigma}{6t \sqrt{l_1}} \left( z - \frac{-l_1 t(3 - l_1) - \mu l_1}{\sqrt{l_1} \sigma} \right) \right)}_{q_1^C} dH(z) \\ &+ \int_{(v_0 - l_1 t(1 - l_1))}^{(v_0 + l_1 t(l_1 + 3))} \underbrace{\frac{\sigma \sqrt{l_1}}{3} \left( z - \frac{-tl_1(3 - l_1) - \mu l_1}{\sigma \sqrt{l_1}} \right)}_{p_1^C} \\ &\quad \underbrace{\left( \frac{\sigma}{6t \sqrt{l_1}} \left( z - \frac{-l_1 t(3 - l_1) - \mu l_1}{\sqrt{l_1} \sigma} \right) \right)}_{q_1^C} \\ &- \underbrace{\frac{\sqrt{l_1} \sigma}{2} \left( z - \frac{-tl_1(1 - l_1) - \mu l_1}{\sqrt{l_1} \sigma} \right)}_{p_1^M - p_0^M} \underbrace{\frac{\sigma}{4t \sqrt{l_1}} \left( z - \frac{-tl_1(1 - l_1) - \mu l_1}{\sqrt{l_1} \sigma} \right)}_{q_1^M} dH(z), \end{aligned}$$

where  $H(z)$  denotes the CDF for  $N(0, 1)$ .

Next, we differentiate the expected difference in the gain from innovation using both the Fundamental Theorem of Calculus and differentiation under the integration sign. All of the Fundamental Theorem of Calculus terms sum up to 0, leaving just the two terms from differentiation under the integration signs:

$$\begin{aligned} & \frac{d}{dl_1} [E(\pi_1^M + \pi_1^M - \bar{\pi}_0) - E(\pi_1^C)] \\ &= \int_{\frac{-l_1 t(3 - l_1) - \mu l_1}{\sqrt{l_1} \sigma}}^{\frac{-l_1 t(1 - l_1) - \mu l_1}{\sqrt{l_1} \sigma}} \underbrace{\frac{\sigma \sqrt{l_1}}{3} \left( z - \frac{-tl_1(3 - l_1) - \mu l_1}{\sigma \sqrt{l_1}} \right)}_{p_1^C} \underbrace{\frac{1}{6tl_1} (3t(1 - l_1) + \mu)}_{\frac{dq_1^C}{dl_1}} dH(z) \\ &+ \int_{\frac{-l_1 t(3 + l_1) - \mu l_1}{\sqrt{l_1} \sigma}}^{\frac{-l_1 t(1 - l_1) - \mu l_1}{\sqrt{l_1} \sigma}} \underbrace{\frac{\sigma \sqrt{l_1}}{3} \left( z - \frac{-tl_1(3 - l_1) - \mu l_1}{\sigma \sqrt{l_1}} \right)}_{p_1^C} \underbrace{\frac{1}{6tl_1} (3t(1 - l_1) + \mu)}_{\frac{dq_1^C}{dl_1}} \\ &- \underbrace{\frac{\sigma \sqrt{l_1}}{2} \left( z - \frac{-tl_1(1 - l_1) - \mu l_1}{\sigma \sqrt{l_1}} \right)}_{p_1^M - p_0^M} \underbrace{\frac{1}{4tl_1} (t(1 - 3l_1) + \mu)}_{\frac{dq_1^M}{dl_1}} dH(z). \end{aligned}$$

The first integral is positive since  $dq_1^C/dl_1 > 0$ . To see that the second integral is also positive, note that  $dq_1^C/dl_1 > dq_1^M/dl_1$  and that, for any  $z \in \left[ \frac{-l_1 t(1 - l_1) - \mu l_1}{\sqrt{l_1} \sigma}, \frac{l_1 t(3 + l_1) - \mu l_1}{\sqrt{l_1} \sigma} \right]$ ,  $p_1^C \geq p_1^M - p_0^M$ . We therefore have

$$\frac{d}{dl_1} [E(\pi_1^M + \pi_1^M - \bar{\pi}_0) - E(\pi_1^C)] > 0. \tag{A.2}$$

The result that  $l_1^C > l_1^M$  then follows from our assumption that the cost function  $c(l_1)$  is sufficiently convex for the objective functions in (6) and (7) to be strictly concave. The final step is to show that such a cost function exists. To this end we show that, for all  $l_1 \in [0, 1/2]$ ,

$$\frac{d^2[E(\pi_1^C)]}{dl_1^2} \text{ and } \frac{d^2[E(\pi_1^M + \pi_1^M - \bar{\pi}_0)]}{dl_1^2}$$

are bounded from above. To this end, we proceed as we did above to find

$$\begin{aligned} \frac{d^2[E(\pi_1^C)]}{dl_1^2} &= \frac{1}{4\sqrt{l_1} l_1} \left[ \frac{1}{3\sigma} l_1 (3t - \mu + 3tl_1)^2 \right. \\ &\quad \left. h\left(-\frac{1}{\sigma \sqrt{l_1}} (\mu l_1 - tl_1(l_1 + 3))\right) \right. \\ &\quad \left. + \frac{1}{9t} \int_{\frac{\sqrt{l_1}}{\sigma}(\mu + t(3 - l_1))}^{-\frac{\sqrt{l_1}}{\sigma}(\mu - t(3 + l_1))} \left( 4t \sqrt{l_1} l_1 (-6t - 2\mu + 3tl_1) \right. \right. \\ &\quad \left. \left. - z\sigma(3tl_1 - (3t + \mu)) \right) dH(z) \right. \\ &\quad \left. - \int_{\frac{\sqrt{l_1}}{\sigma}(\mu - t(3 + l_1))}^{\infty} 8\sqrt{l_1} l_1 t + z\sigma dH(z) \right] \end{aligned}$$

and

$$\begin{aligned} \frac{d^2[E(\pi_0^M + \pi_1^M - \bar{\pi}_0)]}{dl_1^2} &= \frac{1}{4l_1 \sqrt{l_1}} \left[ \frac{1}{4t} \int_{\frac{\sqrt{l_1}}{\sigma}(\mu + t(1 - l_1))}^{-\frac{\sqrt{l_1}}{\sigma}(\mu - t(3 + l_1))} \right. \\ &\quad \left. 4tl_1 \sqrt{l_1} (-2t - 2\mu + 3tl_1) dH(z) \right. \\ &\quad \left. - \frac{1}{4t} \int_{\frac{\sqrt{l_1}}{\sigma}(\mu + t(1 - l_1))}^{-\frac{\sqrt{l_1}}{\sigma}(\mu - t(3 + l_1))} z\sigma(t + \mu + 3tl_1) dH(z) \right. \\ &\quad \left. - \int_{\frac{\sqrt{l_1}}{\sigma}(\mu - t(3 + l_1))}^{\infty} 8t \sqrt{l_1} l_1 + z\sigma dH(z) \right], \end{aligned}$$

where  $h(z)$  denotes the PDF for  $N(0, 1)$ . Notice that both expressions are continuous and finite for  $l_1 = 1/2$ . Since it is also the case that

$$\begin{aligned} \lim_{l_1 \rightarrow 0} \frac{d^2[E(\pi_1^C)]}{dl_1^2} &= \lim_{l_1 \rightarrow 0} \frac{d^2[E(\pi_0^M + \pi_1^M - \bar{\pi}_0)]}{dl_1^2} \\ &= \lim_{l_1 \rightarrow 0} \frac{1}{4\sqrt{l_1}l_1} \left( - \int_0^\infty z\sigma dH(z) \right) = -\infty, \end{aligned}$$

it then follows that both expressions are bounded from above for all  $l_1 \in [0, 1/2]$ .  $\square$

**Proof of Proposition 2.** As noted in the text, we can write the entrant's expected profits as

$$\begin{aligned} E[\pi_1^C + \alpha(\pi_0^M + \pi_1^M - \pi_0^C - \pi_1^C)] \\ = E[(1 - \alpha)\pi_1^C + \alpha(\pi_0^M + \pi_1^M) - \alpha\pi_0^C]. \end{aligned}$$

The proposition then follows from (A.2) and the fact that

$$\frac{d[E(\pi_0^C)]}{dl_1} > 0. \quad (\text{A.3})$$

To show that this inequality holds, we use the expression in Lemma 2 to obtain

$$\begin{aligned} E(\pi_0^C) &= \int_{-\infty}^{v_0 - tl_1(1-l_1) - \bar{p}_0} \bar{\pi}_0 dF(\Delta) \\ &+ \int_{-tl_1(3-l_1)}^{-tl_1(1-l_1) - \bar{p}_0} -\Delta - tl_1(1-l_1) dF(\Delta) \\ &+ \int_{-tl_1(3-l_1)}^{tl_1(3+l_1)} \frac{1}{18tl_1} (\Delta - tl_1(3+l_1))^2 dF(\Delta). \end{aligned}$$

Next, we substitute  $\Delta = \mu l_1 + \sqrt{l_1}\sigma z$  and differentiate with respect to  $l_1$  to obtain

$$\begin{aligned} \frac{d[E(\pi_0^C)]}{dl_1} &= \int_{\frac{-tl_1(3-l_1) - \mu l_1}{\sqrt{l_1}\sigma}}^{\frac{-tl_1(3-l_1) - \mu l_1}{\sqrt{l_1}\sigma}} \frac{\sigma}{2\sqrt{l_1}} \left( -2 \frac{l_1 t(1-2l_1) + l_1 \mu}{\sqrt{l_1}\sigma} - z \right) dH(z) \\ &+ \int_{\frac{-tl_1(3-l_1) - \mu l_1}{\sqrt{l_1}\sigma}}^{\frac{tl_1(3+l_1) - \mu l_1}{\sqrt{l_1}\sigma}} \left( \frac{\sigma}{18t\sqrt{l_1}} (3t(1+l_1) - \mu) \right) \left( \frac{tl_1(3+l_1) - \mu l_1}{\sqrt{l_1}\sigma} - z \right) dH(z). \end{aligned}$$

Notice that at the upper limit  $z = -tl_1(3-l_1) - \mu l_1 / \sqrt{l_1}\sigma$ , the first integrand takes the value  $\frac{1}{2}(t(1+3l_1) - \mu) \geq 0$ , where the inequality follows from  $\mu \in [0, t]$ . Since the first integrand is decreasing in  $z$ , this implies that the first integral is positive. To see that the second integral is also positive, note first that the first factor of the integrand is positive (since  $\mu \in [0, t]$ ). The second factor is decreasing in  $z$  and is equal to zero when  $z$  is at the upper limit  $tl_1(3+l_1) - \mu l_1 / \sqrt{l_1}\sigma$ . This implies that the second integral is also positive which, in turn, proves (A.3).  $\square$

**Proof of Proposition 3.** Consumer surplus is given by

$$\begin{aligned} CS &= \int_{-1/2}^{x(p_1, p_2)} v_0 - ts^2 - p_0 ds + \int_{x(p_1, p_2)}^{1/2} v_0 + \Delta - t(s-l_1)^2 \\ &- p_1 ds, \end{aligned}$$

where  $x(p_1, p_2)$  is the consumer who is indifferent between the two products. We denote consumer surplus for the two pricing games by

$$\begin{aligned} CS^j(z, l_1) &= \int_{-1/2}^{x(p_1^j, p_2^j)} v_0 - ts^2 - p_0^j ds + \int_{x(p_1^j, p_2^j)}^{1/2} v_0 + \mu l_1 \\ &+ \sigma\sqrt{l_1}z - t(s-l_1)^2 - p_1^j ds \text{ for } j = C, M, \end{aligned}$$

where prices  $p_1^M$  and  $p_2^M$  are given by Lemma A.1 and prices  $p_1^C$  and  $p_2^C$  by Lemma A.2,  $z$  denotes a standard normal random variable, and where we replaced the quality difference  $\Delta$  with  $\mu l_1 + \sigma\sqrt{l_1}z$ .

We prove that  $E[CS^C(z, l^C)] \geq E[CS^M(z, l^M)]$  for any  $l^C \in [0, 1/2]$  and  $l^M \in [0, 1/2]$ . To this end, we will show that

$$CS^C(z, l^C) \geq CS^M(z, l^M) \quad (\text{A.4})$$

for all  $z \in \mathbb{R}$ ,  $l^C \in [0, 1/2]$ , and  $l^M \in [0, 1/2]$ .

We start by defining

$$\begin{aligned} X_l(l_1) &\equiv \frac{-tl_1(3-l_1) - \mu l_1}{\sigma\sqrt{l_1}}, \\ X_m(l_1) &\equiv \frac{-tl_1(1-l_1) - \mu l_1}{\sigma\sqrt{l_1}}, \text{ and} \\ X_h(l_1) &\equiv \frac{tl_1(3+l_1) - \mu l_1}{\sigma\sqrt{l_1}}. \end{aligned}$$

Using Lemmas A.1 and A.2, we then have

$$CS^M(z, l_1) = \begin{cases} \frac{1}{6}t + tl_1 & z \geq X_h(l_1) \\ \frac{1}{6}t + \frac{\sigma^2}{16t}(z - X_m(l_1))^2 & X_m(l_1) \leq z \leq X_h(l_1) \\ \frac{1}{6}t & z \leq X_m(l_1) \end{cases}$$

and

$$CS^C(z, l_1) = \begin{cases} v_0 - \frac{1}{12}t + tl_1 & z \geq X_h(l_1) \\ v_0 - \frac{1}{12}t - \frac{1}{3}\sigma\sqrt{l_1}(X_h(l_1) - z) & X_l(l_1) \leq z \leq X_h(l_1) \\ + \frac{\sigma^2}{36t}(z - X_l(l_1))^2 & \\ v_0 - \frac{1}{12}t + \sigma\sqrt{l_1}(z - X_m(l_1)) & X_m(l_1) - \frac{\bar{p}_0}{\sigma\sqrt{l_1}} \leq z \leq X_l(l_1) \\ \frac{1}{6}t & z \leq X_m(l_1) - \frac{\bar{p}_0}{\sigma\sqrt{l_1}}. \end{cases}$$

Suppose first that  $l^M = 0$ . In this case,  $CS^M(z, \frac{1}{2}) = \frac{1}{6}$ , which is weakly less than the lower bound of  $CS^C(z, l^C)$ , given by  $v_0 - \frac{1}{12}t$ .

Suppose now that  $l^M > 0$ . Note that  $CS^M(z, l_1)$  and  $CS^C(z, l_1)$  are both quadratic functions. To prove (A.4), it is, therefore, enough to show that for all  $l^C \in [0, 1/2]$  and any  $l^M \in (0, 1/2]$ ,

$$CS^C(X_h(l^M), l^C) \geq CS^M(X_h(l^M), l^M) \quad (\text{A.5})$$

and

$$CS^C(X_l(l^C), l^C) \geq CS^M(X_l(l^C), l^M). \quad (\text{A.6})$$

We first prove (A.5). Note that the right-hand side of (A.5) is given by  $CS^M(X_h(l^M), l^M) = \frac{1}{6}t + tl^M$ . Suppose first that  $l^C \leq l^M$ . It must then be that  $X_h(l^C) \leq X_h(l^M)$ : at

$z = X_h(l^M)$ , the new product takes the whole market under either structure. We then have

$$CS^C(X_h(l^M), l^C) = v_0 - \frac{1}{4}t + \frac{1}{6}t + tl^C \geq t + \frac{1}{6}t + tl^C,$$

where the second inequality follows from the covered-market assumption  $v_0 \geq 5t/4$ . We therefore get the desired inequality,

$$CS^C(X_h(l^M), l^C) \geq t + \frac{1}{6}t + tl^C > \frac{1}{6}t + tl^M = CS^M(X_h(l^M), l^M),$$

for all  $l^C \leq l^M$  and any  $l^M \in (0, 1/2]$ .

Next, suppose that  $l^C \geq l^M$ . We can then write

$$\begin{aligned} & CS^C(X_h(l^M), l^C) - CS^M(X_h(l^M), l^M) \\ &= \left( v_0 - \frac{1}{12}t - \frac{1}{3}\sigma\sqrt{l^C}(X_h(l^C) - X_h(l^M)) + \frac{\sigma^2}{36t}(X_h(l^M) - X_l(l^C))^2 \right) - \left( \frac{1}{6}t + tl^M \right) \\ &\geq \left( v_0 - \frac{1}{12}t - \frac{1}{3}\sigma\sqrt{l^C}X_h(l^C) + \frac{\sigma^2}{36t}X_h(l^M)^2 \right) - \left( \frac{1}{6}t + tl^M \right) \\ &\geq t \left( 1 - \frac{1}{3}(l^C(3+l^C) - \frac{\mu}{t}l^C) - l^M \left( 1 - \frac{1}{36}((3+l^M) - \frac{\mu}{t})^2 \right) \right) \\ &\geq t \left( 1 - \frac{1}{3}(l^C(3+l^C)) - l^M \left( 1 - \frac{1}{36}((3+l^M) - 1)^2 \right) \right) \\ &\geq t \left( 1 - \frac{1}{3} \left( \frac{1}{2} \left( 3 + \frac{1}{2} \right) \right) - \frac{1}{2} \left( 1 - \frac{1}{36} \left( \left( 3 + \frac{1}{2} \right) - 1 \right)^2 \right) \right) = \frac{t}{288} > 0, \end{aligned}$$

where the first inequality follows from replacing  $X_h(l^C) - X_h(l^M)$  with  $X_h(l^C)$  and  $X_h(l^M) - X_l(l^C)$  with  $X_h(l^M)$ ; the second follows from the covered-market assumption that  $v_0 \geq 5t/4$ ; the third follows from replacing the first  $\mu/t$  with 0 and the second one with 1; and the fourth from replacing  $l^C$  and  $l^M$  with  $1/2$ . We therefore get the desired inequality for  $CS^C(X_h(l^M), l^C) \geq CS^M(X_h(l^M), l^M)$  for all  $l^C \geq l^M$ .

Next, we prove (A.6). Note that the right-hand side of (A.6) is maximized for  $l^M = \frac{1}{3}(1 + \frac{\mu}{t})$ . It is therefore sufficient to show that

$$CS^C(z = X_l(l^C), l^C) \geq CS^M\left(X_l(l^C), \frac{1}{3}\left(1 + \frac{\mu}{t}\right)\right) \text{ for all } l^C \in [0, 1/2].$$

This inequality holds trivially for any  $l^C$  such that  $X_l(l^C) \leq X_m(\frac{1}{3}(1 + \frac{\mu}{t}))$  since, in this case,  $CS^M(X_l(l^C), \frac{1}{3}(1 + \frac{\mu}{t})) = \frac{1}{6}t$  which is the lower bound. It is therefore sufficient to focus on values of  $l^C$  such that

$$X_l(l^C) \geq X_m\left(\frac{1}{3}\left(1 + \frac{\mu}{t}\right)\right), \tag{A.7}$$

or, equivalently,

$$\sqrt{l^C}\left(3 - l^C + \frac{\mu}{t}\right) \leq \sqrt{\frac{1}{3}\left(1 + \frac{\mu}{t}\right)}\frac{2}{3}\left(1 + \frac{\mu}{t}\right).$$

In this case, we have

$$\begin{aligned} & CS^C(X_l(l^C), l^C) - CS^M\left(X_l(l^C), \frac{1}{3}\left(1 + \frac{\mu}{t}\right)\right) \\ &= v_0 - \frac{1}{4}t + \sigma\sqrt{l^C}(X_l(l^C) - X_m(l^C)) - \frac{\sigma^2}{16t}\left(X_l(l^C) - X_m\left(\frac{1}{3}\left(1 + \frac{\mu}{t}\right)\right)\right)^2 \\ &\geq t + \sigma\sqrt{l^C}(X_l(l^C) - X_m(l^C)) - \left(\frac{\sigma^2}{16t}\left(X_l(l^C) - X_m\left(\frac{1}{3}\left(1 + \frac{\mu}{t}\right)\right)\right)^2\right) \\ &= t \left[ 1 - 2l^C - \frac{1}{16}\left(\sqrt{\frac{1}{3}\left(1 + \frac{\mu}{t}\right)}\frac{2}{3}\left(1 + \frac{\mu}{t}\right) - \sqrt{l^C}\left(3 - l^C + \frac{\mu}{t}\right)\right)^2 \right] \\ &\geq t \left[ 1 - 2l^C - \frac{1}{16}\left(\frac{4}{9}\sqrt{2}\sqrt{3} - \sqrt{l^C}\left(3 - l^C\right)\right)^2 \right], \end{aligned}$$

where the first inequality follows from the covered-market assumption  $v_0 \geq 5t/4$ , and the second inequality follows from setting  $\mu/t$  equal to 0 in the first term and equal to 1 in the second. The last inequality is positive for all  $l^C \leq 0.486$ . The only remaining thing to do is to show that  $l^C \geq 0.486$ , violates the constraint (A.7). Since the left-hand side of the constraint is increasing in  $l^C$  it is sufficient to show that it is violated for  $l^C = 0.486$ , that is,

$$\sqrt{0.486}\left((3 - 0.486) + \frac{\mu}{t}\right) - \sqrt{\frac{1}{3}\left(1 + \frac{\mu}{t}\right)}\frac{2}{3}\left(1 + \frac{\mu}{t}\right) > 0.$$

This inequality holds for any  $\frac{\mu}{t} \leq 1$ , which completes the proof.  $\square$

### Endnotes

<sup>1</sup> Our definition of innovation follows that of Rogers (1962, p. 475): “An idea, practice, or object that is perceived as new by an individual or other unit of adoptions.” It does not require that the outcome be a success.

<sup>2</sup> In this way we differ from the literature on the direction of innovation. The representation we introduce can be interpreted as how far along a given direction an innovator wishes to push. We discuss connections to the literature momentarily.

<sup>3</sup> One might also think of this as the extensive versus the intensive margin of innovation.

<sup>4</sup> See Philippon (2019) for a book-length treatment with particular emphasis on the diverging paths of the United States and the European Union.

<sup>5</sup> In practice, this relationship may not be perfectly correlated, and Bloom et al. (2013) provide evidence suggesting how it varies across technologies and industries. Exploring how this affects innovation and competition in our setting, particularly when the degree of correlation is itself a choice variable, is beyond the scope of this paper, although an interesting direction for future work.

<sup>6</sup> This typology builds, in turn, on the even more classic exploration versus exploitation trade-off and applies it across firms.

<sup>7</sup> Fitzgerald et al. (2021) recently provided evidence that incremental innovations are under-appreciated by the market relative to bolder innovations.

<sup>8</sup> Additional models include Bryan and Hovenkamp (2020a), Gilbert (2019), Letina and Schmutzler (2019), and Cabral (2018).

<sup>9</sup> See Gans (2016) for a thorough review. Another difference is that our model is static whereas the Christensen story is inherently dynamic. Adner and Zemsky (2005) provide a rational model consistent with the dynamic path of disruption described in Christensen (1997).

<sup>10</sup> This line of work is connected to an earlier literature on competition-destroying acquisitions and their impact on entry decisions (Gilbert and Newbery 1982, Katz and Shapiro 1985, Rasmusen 1988).

<sup>11</sup> See also Bryan and Hovenkamp (2020b), Gilbert (2019), Letina and Schmutzler (2019), Cabral (2018), Letina et al. (2020), Moraga-Gonzalez et al. (2019), and Kamepalli et al. (2021).

<sup>12</sup> The microfoundation for this formulation is that there exists a mapping from technology/product space to outcomes, and that this mapping is the realized path of a Brownian motion anchored at  $(0, v_0)$ , with drift  $\mu$  and variance  $\sigma^2$  (see Callander and Matouschek 2019 for more elaboration).

<sup>13</sup> Meagher and Zauner (2004) extend the Hotelling model to allow for uncertainty over demand. In their model, uncertainty is

exogenous, common across firms, and independent of the firms' strategies, which is very different from uncertainty due to innovation.

<sup>14</sup> Adner and Zemsky (2005) offer the interpretation of the Hotelling line as representing technological distance. Implicit in this formulation is that consumer preference and uncertainty over technology is aligned. This alignment will be less than perfect in practice.

<sup>15</sup> This implies the market is "covered." In addition to tractability, this assumption is the best case for the Arrow replacement effect as all sales of the incumbent must cannibalize the existing product, adding robustness to our reversal result (Proposition 2).

<sup>16</sup> The exception is for a nuisance product. In that case,  $\pi_0^C$  is the only term that varies in  $l_1$  and, in this case only, it decreases in  $l_1$ . Note that this term cancels out and the net effect on  $(G^C - G^M)$  is zero.

<sup>17</sup> We are tempted to refer to this as a "seppuku acquisition," in which the incumbent acquires the new firm only to kill itself, although formally, because  $v_0 > 0$ , the existing product is only shut down when it would have zero market share even in a competitive market.

<sup>18</sup> It is for this reason that we do not explore the welfare implications of the propositions. With a covered market, the lower prices that competition brings about do not increase welfare. Instead, welfare depends only on the price differentials and is maximized if the prices for both products are equal. A welfare comparison of monopoly versus competition boils down, therefore, to a comparison of the price differentials in Figure 1.

<sup>19</sup> See Gans (2011) for an argument that static analyses are often sufficient for, and at least more predictive of, market behavior in many environments.

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