What do Financial markets say about the exchange rate?*
Not all that much...

Mikhail Chernov & Valentin Haddad & Oleg Ithskhoki
discussion by H. Lustig

Summary of CHI (2023)

- Very little we can learn about exchange rates from (largely uninformative) financial market moments.
- Especially if you rule out completely implausible asset market structures.
- Finance is a sideshow when you're trying to understand exchange rates.
- Commonly held view in macro.

What do Financial markets say about the exchange rate?*
Quite a lot!

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My Take on This Paper

- Lots we can learn (and have learned) about exchange rates from (highly informative) financial market moments.
- Especially if you have to rule out completely plausible asset market structures to match moments.
- Agree with most of the equations.
- Don't quite agree with the words.
- Provocative early-stage paper; still converging.

Complete Markets: Three Major FX Puzzles

- Take your favorite (rep. agent) SDF, e.g. $m_{t+1} = \beta(\frac{c_{t+1}}{c_t})^{-\gamma}$.
- With *Complete Markets*, Exchange rates are shock absorbers: $\Delta s_{t+1} = m_{t+1} - m^*_{t+1}$.

1. **Volatility puzzle**: Brandt, Cochrane, and Santa-Clara (2006)

$$\begin{aligned} var_t(\Delta s_{t+1}) &= var_t(m^*_{t+1}) + var(m_{t+1}) \\ &- 2\rho_t(m_{t+1}, m^*_{t+1}) std_t(m_{t+1}) std_t(m^*_{t+1}). \end{aligned}$$

2. **Counter-cyclicality/Backus-Smith puzzle**: Kollmann (1991), Backus and Smith (1993)

$$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0.$$

3. **Risk premium puzzle** (Forward premium / UIP puzzle): Tryon (1979), Hansen and Hodrick (1980), Fama (1984), Backus, Foresi and Telmer (2001)

Shutting Down Markets: Only 4 Bond Euler Equations

- 4 Equations considered by Lustig and Verdelhan (2019)
- Domestic investors (with access to bond markets) face Euler equations for domestic and foreign currency bonds:

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1} + r_t)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right].
\end{aligned}
$$

- Foreign investors (with access to bond markets) face Euler equations for foreign and domestic currency bonds:

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[\exp(m^*_{t,t+1} + r_t^*)\right], \\
1 &= \mathbb{E}_t \left[\exp(m^*_{t,t+1} + \Delta s_{t+1} + r_t)\right].
\end{aligned}
$$

- Does not specify the other securities traded across borders/currencies (*Incomplete Markets*).

Partially Integrated Markets

- CHI (2023) label this *special case* with 4 bond Euler equations *Partially Integrated Markets*:

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1} + r_t)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1}^* + r_t^*)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1}^* + \Delta s_{t+1} + r_t)\right].
\end{aligned}
$$

  - Does not require all *households* to trade foreign currency and domestic currency bonds, only some *investors* .
    - Including levered financial institutions (as in He and Krishnamurthy (2013) and Kelly, He, Manela (2017)).
  - Does not require *any investors* to trade foreign equities or any other foreign securities.
  - Covers large class of 2-country International Business Cycle Models. (Chari, Kehoe, and McGrattan (2002)).

Incomplete Markets: Backus-Smith Puzzle

- FX risk premium demanded by domestic investor:

  $$(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) = -cov_t(m_{t,t+1}, -\Delta s_{t+1}).$$

- FX risk premium demanded by foreign investor:

  $$(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) = -cov_t(m^*_{t,t+1}, \Delta s_{t+1}).$$

- Just add 2 FX risk premium equations to obtain:

  $$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0.$$

- 4 Euler equations $\rightarrow$ Conditionally **counter-cyclical exchange rates** (see Lustig and Verdelhan (2019)).
  - Same expression as in *Complete Markets*

Exchange Rates are Traded..

- Project the SDF onto the space of traded assets:

$$\begin{aligned}\lambda^*_{t+1} &= proj(m^*_{t+1}|X^*) = \mathbb{E}_t(m^*_{t+1}) + \beta^*_t\left(\Delta s_{t+1} - \mathbb{E}(\Delta s_{t+1})\right),\\ \lambda_{t+1} &= proj(m_{t+1}|X) = \mathbb{E}_t(m_{t+1}) + \beta_t\left(-\Delta s_{t+1} + \mathbb{E}(\Delta s_{t+1})\right).\end{aligned}$$

  - Projection coefficients with $\beta_t + \beta^*_t = -1$

$$\beta_t = \frac{cov_t(-\Delta s_{t+1}, m_{t+1})}{var_t(\Delta s_{t+1})} \leq 0, \beta^*_t = \frac{cov_t(\Delta s_{t+1}, m^*_{t+1})}{var_t(\Delta s_{t+1})} \leq 0$$

- Exchange rates are given by $\Delta s_{t+1} = \lambda_{t+1} - \lambda^*_{t+1}$;
  - We can shrink $var_t(\Delta s_{t+1})$ (see Lustig and Verdelhan (2019)).
  - At cost of shrinking FX Risk premiums :

$$(r^*_t - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) = -\beta_t var_t(\Delta s_{t+1}).$$

- Same result for covariance with these MV SDFs: $cov_t(\lambda_{t,t+1} - \lambda^*_{t,t+1}, \Delta s_{t+1}) = (\beta_t + \beta^*_t) var_t(\Delta s_{t+1}) = var_t(\Delta s_{t+1})$.

Spanning

- Project the SDF onto the space of traded assets:

$$\begin{aligned}\lambda^*_{t+1} &= proj(m^*_{t+1}|X^*) = \mathbb{E}_t(m^*_{t+1}) + \beta^*_t\left(\Delta s_{t+1} - \mathbb{E}(\Delta s_{t+1})\right),\\ \lambda_{t+1} &= proj(m_{t+1}|X) = \mathbb{E}_t(m_{t+1}) + \beta_t\left(-\Delta s_{t+1} + \mathbb{E}(\Delta s_{t+1})\right).\end{aligned}$$

- Projection coefficients with $\beta_t + \beta^*_t = -1$
- Exchange rates are given by $\Delta s_{t+1} = \lambda_{t+1} - \lambda^*_{t+1}$

- Not sure market structure can be **tested** by checking whether exchange rates are spanned.
  - Suppose we trade foreign and domestic risk-free bonds.
  - Then all this says is that the exchange rate explains itself.
  - Exchange rates do not have to spanned by returns on other assets, like bond returns, because we're trading the risk-free.

Back to Meese-Rogoff and Exchange Rate Disconnect

- *Complete Markets*: the **unconditional** exchange rate cyclicality satisfies $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var(\Delta s_{t+1}) > 0.$

- *Incomplete Markets*: A necessary condition for $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$ (Atkeson, Jiang, Krishnamurthy and Lustig (2023)):

$$\sqrt{\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{1}{\sqrt{R^2}}.$$

  - b is the Fama slope coefficient in $\Delta s_{t+1} = a + b(f_t - s_t) + \varepsilon_{t+1}$,
  - $R^2 = var(\mathbb{E}_t[\Delta s_{t+1}])/var(\Delta s_{t+1})$ in a forecasting regressions.

- as $R^2 \to 0$ (Meese and Rogoff (1983)), this becomes an impossibility result.

Four Major International Finance Puzzles

1. **Volatility puzzle**: Brandt, Cochrane, and Santa-Clara (2006)
2. **Counter-cyclicality/Backus-Smith puzzle** (correlation puzzle, exchange rate disconnect): Kollmann (1991), Backus and Smith (1993)
3. **Risk premium puzzle** (Forward premium / UIP puzzle): Tryon (1979), Hansen and Hodrick (1980), Fama (1984)
4. **Exchange Rate Disconnect** : Meese and Rogoff (1984)

What do Financial Markets say about Exchange Rates?

- Can't square these 4 bond Euler equations with other moments of FX (listed in (1-4)):

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[\exp(m_{t,t+1} + r_t)\right], \\
1 &= \mathbb{E}_t\left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right], \\
1 &= \mathbb{E}_t\left[\exp(m^*_{t,t+1} + r_t^*)\right], \\
1 &= \mathbb{E}_t\left[\exp(m^*_{t,t+1} + \Delta s_{t+1} + r_t)\right].
\end{aligned}
$$

- We can't have domestic and foreign investors do the FX carry trade.
  - Like restating the equity premium puzzle by saying that investors cannot both invest in equities and bonds.
- Financial market moment conditions seem quite (too) informative about exchange rates.

Segmented Bond Markets

- Need **segmentation** of foreign from domestic currency bond markets (and other domestic securities markets) for **all investors**, including financial institutions.

    1. **FX Markets Intermediated**: Domestic investors with IMRS m cannot directly access foreign currency bond market. (Gabaix-Maggiori (2015); Itskhoki and Mukhin (2021);Fukui, Nakamura, and Steinsson (2023), Greenwood-Hanson-Sunderam-Stein (2020); Gourinchas, Ray and Vayanos (2020))
    2. **Shutting down more markets**: Investors trade only bonds denominated in global numeraire. (Corsetti, Dedola, and Leduc (2008), Pavlova and Rigobon (2012))
    3. **Transaction Costs**: (Alvarez, Atkeson, and Kehoe (2002, 2009)).

4 Bond Euler Equations with Wedges.

- Some domestic investors can invest in foreign currency bonds.
- **Segmentation** introduces wedges in domestic investor's Euler equation:

$$\begin{aligned} 1 &= \mathbb{E}_t \left[\exp(m_{t+1} + r_t)\right], \\ \exp(\xi_t) &= \mathbb{E}_t \left[\exp(m_{t+1} - \Delta s_{t+1} + r_t^*)\right], \\ 1 &= \mathbb{E}_t \left[\exp(m_{t+1}^* + r_t^*)\right], \\ \exp(\xi_t^*) &= \mathbb{E}_t \left[\exp(m_{t+1}^* + \Delta s_{t+1} + r_t)\right]. \end{aligned}$$

- Interpretation of these wedges (not really UIP shocks):
  1. Transaction costs in foreign FX bonds ($\xi_t > 0, \xi_t^* > 0$)
  2. **Home currency bias** ($\xi_t > 0, \xi_t^* > 0$)
     - Investors strictly prefer bonds denominated in domestic currency, except USD (Maggiori, Neiman, Schreger (2021)).
  3. Convenience yields from foreign FX bonds ($\xi_t < 0, \xi_t^* < 0$)

4 Bond Euler Equations with Wedges.

- **Segmentation** introduces wedges in domestic investor's Euler equation:

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[\exp(m_{t+1} + r_t)\right], \\
\exp(\xi_t) &= \mathbb{E}_t\left[\exp(m_{t+1} - \Delta s_{t+1} + r_t^*)\right], \\
1 &= \mathbb{E}_t\left[\exp(m_{t+1}^* + r_t^*)\right], \\
\exp(\xi_t^*) &= \mathbb{E}_t\left[\exp(m_{t+1}^* + \Delta s_{t+1} + r_t)\right].
\end{aligned}
$$

- Interpretation of these wedges:
  1. Transaction costs in foreign FX bonds ($\xi_t > 0, \xi_t^* > 0$)
  2. Home currency bias ($\xi_t > 0, \xi_t^* > 0$)
  3. **Convenience yields from foreign FX bonds** ($\xi_t < 0, \xi_t^* < 0$)
     - Investors strictly prefer USD denominated safe assets (U.S. Treasurys) (Jiang, Krishnamurthy, Lustig (2021)).

Backus-Smith Puzzle

- FX Risk premium demanded by domestic and foreign investors:

$$
\begin{aligned}
(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}, -\Delta s_{t+1}) \\
&+ \xi_t. \\
(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) &= -cov_t(m^*_{t,t+1}, \Delta s_{t+1}) \\
&+ \xi_t^*.
\end{aligned}
$$

- Just add 2 equations to obtain:

$$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) - (\xi_t + \xi_t^*).$$

- 4 Euler equations with wedges $\rightarrow$ Conditionally **pro-cyclical** exchange rates iff $(\xi_t + \xi_t^*) > var_t(\Delta s_{t+1})$

1. **Transaction costs** ($\xi_t > 0, \xi_t^* > 0$) ✓
2. **Home currency bias** ($\xi_t > 0, \xi_t^* > 0$) ✓
3. **Convenience yields** from foreign currency bonds ($\xi_t < 0, \xi_t^* < 0$)

Home Bias

- *Wedges*: A necessary condition for $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$ (Atkeson, Jiang, Krishnamurthy and Lustig (2023)):

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])} \geq \frac{1}{R^2}\left(1 - \frac{\mathbb{E}(\xi^*_t + \xi_t)}{var(\Delta s_{t+1})}\right).$$

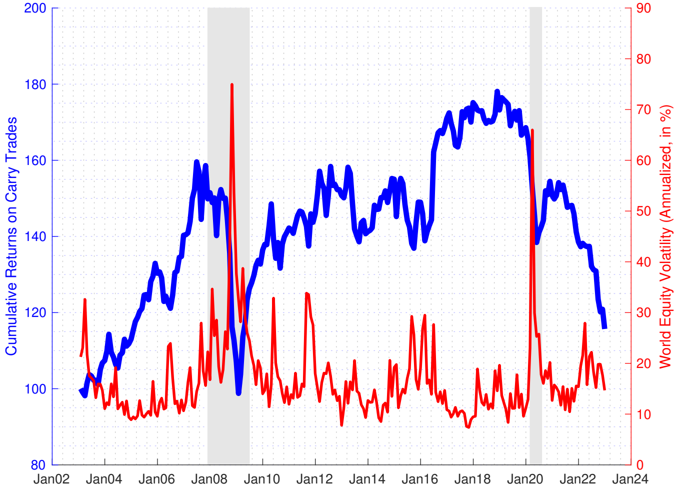
  - b is the Fama slope coefficient in $\Delta s_{t+1} = a + b(f_t - s_t) + \varepsilon_{t+1}$, and
  - $R^2 = var(\mathbb{E}_t[\Delta s_{t+1}])/var(\Delta s_{t+1})$ in forecasting regression.

- Even as $R^2 \to 0$ (Meese and Rogoff (1983)), we can satisfy the condition with a large home currency bias $\mathbb{E}(\xi^*_t + \xi_t) > 0$.
  - Only first moment of U.I.P. wedges matters.

Everything is Connected

1. **Volatility puzzle**: Brandt, Cochrane, and Santa-Clara (2006)
2. **Counter-cyclicality/Backus-Smith puzzle** (correlation puzzle, exchange rate disconnect): Kollmann (1991), Backus and Smith (1993)
3. **Risk premium puzzle** (Forward premium / UIP puzzle): Tryon (1979), Hansen and Hodrick (1980), Fama (1984)
4. **Exchange Rate Disconnect** : Meese and Rogoff (1984)
5. **Home Bias** : French and Poterba (1991), Tesar and Werner (1998), Maggiori, Neiman and Schreger (2020)

Currency Risk Premia are Real (11/1983–2/2021, sample of developed countries)



Conclusion

- Provocative paper.
- Lots we have learned from financial markets about exchange rates.

Limited Participation, No Euler Equation Wedges

- Limited Participation SDF m_{t+1} is the IMRS of small pool of investors active in asset markets at home and abroad.
- Exchange rates are shock absorbers: $\Delta s_{t+1} = m_{t+1} - m^*_{t+1}$.

1. **Volatility puzzle**:

$$\begin{aligned} var_t(\Delta s_{t+1}) &= var_t(m^*_{t+1}) + var(m_{t+1}) \\ &- 2\rho_t(m_{t+1}, m^*_{t+1}) std_t(m_{t+1}) std_t(m^*_{t+1}). \end{aligned}$$

   $\rho_t(m_{t+1}, m^*_{t+1}) \to 1$

2. **Counter-cyclicality/Backus-Smith puzzle**:

$$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0.$$

   $m_{t,t+1}$ disconnected from domestic business cycle.

3. **Risk premium puzzle** (Forward premium / UIP puzzle)

Hodrick Projection Argument with Lustig-Verdelhan Wedge

- lowercase denotes logs
- When projecting the log foreign SDF onto the space of internationally traded assets, we get the following:

$$\lambda^*_{t+1} = proj(m^*_{t+1}|X) = E_t(m^*) + \beta^* \left(-\Delta s_{t+1} + E(\Delta s_{t+1})\right)$$

- we introduce a wedge:

$$\Delta s_{t+1} = m^*_{t+1} - m_{t+1} + \eta_{t+1}$$

- projection coefficient:

$$\beta^*(\eta) = -\frac{cov_t(\Delta s_{t+1}, m^*_{t+1})}{var_t(\Delta s_{t+1})}$$

Hodrick Projection Argument with Lustig-Verdelhan Wedge

- projection coefficient:

$$\begin{aligned}\beta(\eta) &= \frac{cov_t(m^*_{t+1} - m_{t+1} + \eta_{t+1}, m_{t+1})}{var_t(\Delta s_{t+1})} \\ &= \frac{cov_t(m^*_{t+1} - m_{t+1}, m_{t+1})}{var_t(\Delta s_{t+1})} + \frac{-\mu_{t,\eta} + \frac{1}{2} var_t\left(\eta_{t+1}\right)}{var_t(\Delta s_{t+1})}\end{aligned}$$

  where we have used our second condition in prop 1.
- note that $\beta(\eta) \leq 0$
- the wedge does not drop out

Hodrick Projection Argument with Lustig-Verdelhan Wedge

- projection coefficient:

$$\begin{aligned}\beta^*(\eta) &= -\frac{cov_t(m^*_{t+1} - m_{t+1} + \eta_{t+1}, m^*_{t+1})}{var_t(\Delta s_{t+1})} \\ &= -\frac{cov_t(m^*_{t+1} - m_{t+1}, m^*_{t+1})}{var_t(\Delta s_{t+1})} - \frac{-\mu_{t,\eta} - \frac{1}{2} var_t\left(\eta_{t+1}\right)}{var_t(\Delta s_{t+1})}\end{aligned}$$

  where we have used our first condition in prop 1.
- the wedge does not drop out
- note that $\beta^*(\eta) \leq 0$
- note that $\beta^*(\eta) + \beta^*(\eta) = -1$, as in Bob's derivation.

Lognormal Case

- Assumptions:
  - conditional joint log normality;
  - investors only trade risk-free bonds.

Result

Fix (m, m^*) and the risk-free rates. We can construct a new exchange rate $\Delta s_{t+1} = m^*_{t+1} - m_{t+1} + \eta_{t+1}$ by adding a wedge η_{t+1} such that

$$
\begin{aligned}
covar_t\left(m^*_{t+1}, \eta_{t+1}\right) &= -\mu_{t,\eta} - \frac{1}{2} var_t\left(\eta_{t+1}\right), \\
covar_t\left(m_{t+1}, \eta_{t+1}\right) &= -\mu_{t,\eta} + \frac{1}{2} var_t\left(\eta_{t+1}\right).
\end{aligned}
$$

and some additional restrictions on $\mu_{t,\eta} = E_t\left(\eta_{t+1}\right)$.