

Debt And Deficits: Fiscal Analysis With Stationary Ratios

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Fiscal Sustainability: Forward-looking Approach

- ▶ Apply standard asset pricing machinery ([Campbell-Shiller](#) decomposition) to a macro question (fiscal sustainability)
- ▶ Market value of debt/output ratio is a valuation ratio (akin to price/dividend ratio)
- ▶ What drives a high market value of U.S. debt/output ratio back down?
 1. **Discount rates:** higher debt/output ratio predicts lower future real returns (and/or higher future output growth) (lower $r - g$) ([Blanchard, 2019](#); [Furman and Summers, 2020](#); [Cochrane, 2021a](#)) .
 2. **Cash flows:** higher debt/output ratio predicts higher future surpluses ([Bohn, 1998](#); [Cochrane, 2020](#)).
 3. **Nothing at all:** higher debt/output ratio predicts higher future debt/output ratio (close to unit root).

Campbell-Shillerize it.

- ▶ Log-linearized return equation implied by the government budget constraint:

$$\tilde{r}_{t+1} = r_{t+1} - \pi_{t+1} - x_{t+1} = \rho v_{t+1} - v_t + s_{t+1},$$

where $\rho = \exp(-(r - x - \pi))$ is a constant, v_t is log of MV debt/output ratio, and $s_{t+j} = sy_{t+j}/e^v$ is a scaled measure of surplus/output.

(see [Gourinchas and Rey, 2007](#); [Berndt, Lustig, and Yeltekin, 2012](#); [Cochrane, 2021a](#))

- ▶ By iterating this forward T times and taking expectations, we obtain the debt valuation equation:

$$v_t = \mathbb{E}_t \sum_{j=1}^T \rho^{j-1} (s_{t+j} - \tilde{r}_{t+j}) + \mathbb{E}_t \rho^T v_{t+T}.$$

- ▶ Debt/output ratio varies because it either predicts future surpluses, future returns, or the future debt/output ratio:

$$\text{var}(v_t) = \text{cov} \left(\sum_{j=1}^T \rho^{j-1} s_{t+j}, v_t \right) - \text{cov} \left(\sum_{j=1}^T \rho^{j-1} \tilde{r}_{t+j}, v_t \right) + \text{cov}(v_t, \rho^T v_{t+T}).$$

Cash Flows or Discount Rates

- ▶ Earlier work:
 - ▶ [Bohn \(1998\)](#), studying a sample that ends in the mid-1990s, finds evidence that the primary surplus increases when the debt/output ratio is high
 - ▶ [Cochrane \(2021a,b\)](#) finds evidence that the debt/output ratio predicts lower *nominal* returns on the government debt portfolio
 - ▶ [Jiang, Lustig, VanN. and Xiaolan \(2022\)](#): no evidence that the debt/output ratio predicts surpluses or *real growth-adjusted* returns ; debt/output ratio is very persistent

$$1 = \frac{\text{cov}\left(\sum_{j=1}^T \rho^{j-1} s_{t+j}, v_t\right)}{\text{var}(v_t)} - \frac{\text{cov}\left(\sum_{j=1}^T \rho^{j-1} \tilde{r}_{t+j}, v_t\right)}{\text{var}(v_t)} + \frac{\text{cov}(v_t, \rho^T v_{t+T})}{\text{var}(v_t)}.$$

- ▶ [Key observation in JLVX \(2022\)](#): Large small-sample bias ([Stambaugh, 1999](#)) in the slope coefficients of the return and surplus predictability regressions due to:
 1. High **persistence** of the debt/output ratio (the predictor is close to a unit root)
 2. High **correlation** between the innovations to the predictor and the predicted variables

The Dogs that Didn't Bark (JLVX (2022))

- ▶ Campbell-Shiller decomposition of the U.S. debt/output ratio :
 1. **Discount rates:** No evidence that the debt/output ratio predicts real growth-adjusted returns. ✗
 2. **Cash flows:** No evidence that the debt/output ratio predicts surpluses. ✗
 3. **Residual:** the debt/output ratio predicts higher future debt/output ratio ✓

$$1 = \frac{\text{cov}\left(\sum_{j=1}^T \rho^{j-1} s_{t+j}, v_t\right)}{\text{var}(v_t)} - \frac{\text{cov}\left(\sum_{j=1}^T \rho^{j-1} \tilde{r}_{t+j}, v_t\right)}{\text{var}(v_t)} + \frac{\text{cov}(v_t, \rho^T v_{t+T})}{\text{var}(v_t)}.$$

⇒ Excess smoothness: Bond prices today not responsive to news about future macro fundamentals

This paper

- ▶ Overview:
 - ▶ [Bohn \(1998\)](#), studying a sample that ends in the mid-1990s, finds evidence that the primary surplus increases when the debt/output ratio is high
 - ▶ [Cochrane \(2021a,b\)](#) finds evidence that the debt/output ratio predicts lower *nominal* returns on the government debt portfolio
 - ▶ [Jiang, Lustig, VanN. and Xiaolan \(2022\)](#): no evidence that the debt/output ratio predicts surpluses or *real growth-adjusted* returns; debt/output ratio is very persistent
 - ▶ [Campbell, Gao and Martin \(2023\)](#): debt/output ratio has unit root; no evidence that the surplus/debt ratio predicts *real* returns; some evidence that surplus/debt ratio predicts tax growth
- ▶ [Key observation in CGM \(2023\)](#):
 1. The market value of debt/output ratio has **unit root**, but the surplus/debt ratio is stationary (the CGM predictor is the surplus/debt ratio)
 2. The surplus/debt ratio predicts **cash flows**, but not returns.

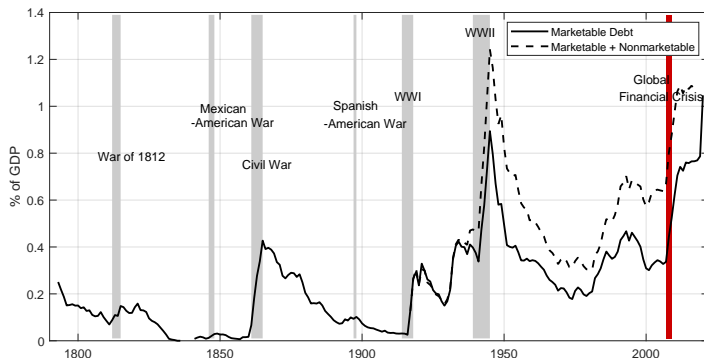
Related Literature

- ▶ **Stock return predictability** (Campbell and Thompson, 2007; Cochrane, 2008; Binsbergen and Koijen, 2010; Goyal and Welch, 2005; Golez and Koudijs, 2018):
 - ▶ Discount rates on stocks are remarkably volatile (Hansen and Jagannathan, 1991),
 - ▶ Valuation of stocks seems excessively volatile compared to its fundamentals (LeRoy and Porter, 1981; Shiller, 1981),
 - ▶ High valuations imply low future returns (mean reversion in valuation ratios),
- ▶ **Bond return predictability:** (Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005; Ludvigson and Ng, 2009; Cochrane, 2011) ,
 - ▶ Individual bond return predictability,
 - ▶ For entire bond portfolio
 - ▶ **JLVX (2022)**: high valuations do not imply low future returns (little mean reversion in sample in valuation ratios); Valuation of bonds seems too smooth compared to its fundamentals
 - ▶ **CGM (2023)**: high valuations do not imply low future returns but imply larger future tax revenue growth ; mean reversion in different valuation ratio.

Key CGM (2023) Findings

1. **Traditional valuation ratio:** The market value of debt/output ratio has **unit root**.

- ▶ Debt/output ratio in the U.S. is highly persistent. ✓

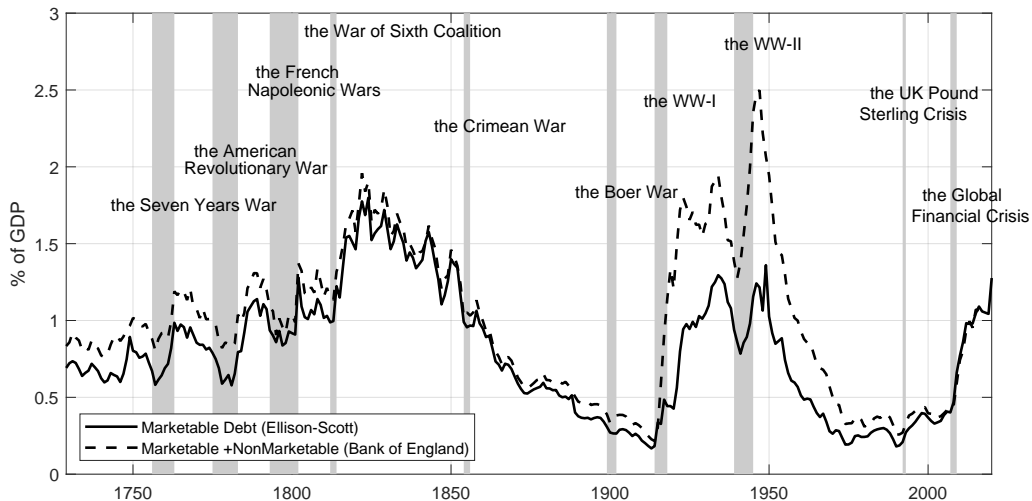


2. **New valuation ratio:** The surplus/debt ratio is stationary (the CGM (2023) predictor is the surplus/debt ratio)

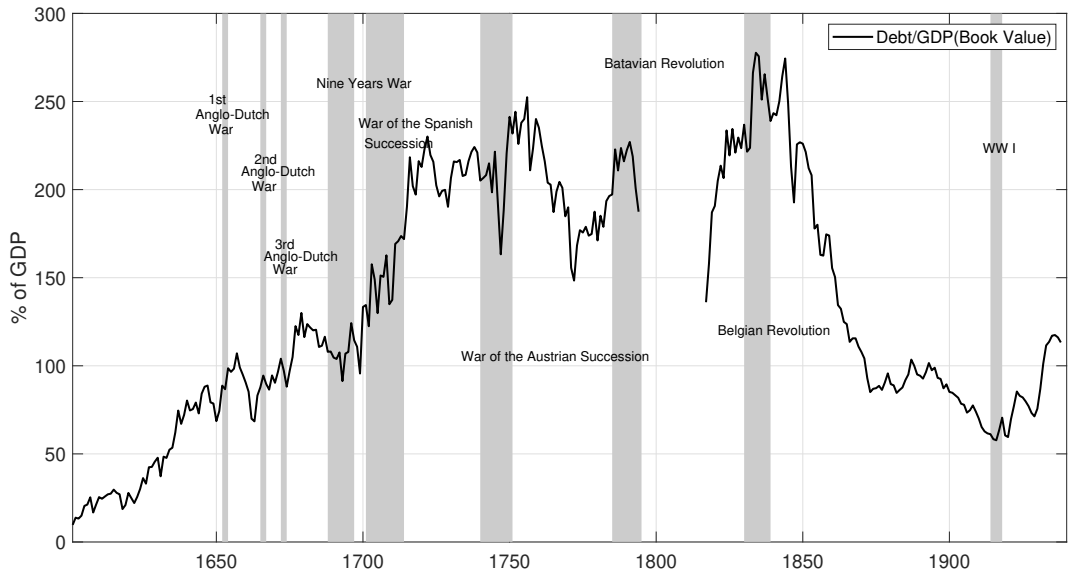
Key CGM (2023) Findings

1. **Traditional valuation ratio:** The market value of debt/output ratio has **unit root**.
 - ▶ Debt/output ratio in the U.S. is highly persistent. ✓
 - ▶ Unit root hard to reconcile with historical evidence; more consistent with slow mean-reversion in debt/output.
2. **New valuation ratio:** The surplus/debt ratio is stationary (the CGM (2023) predictor is the surplus/debt ratio)

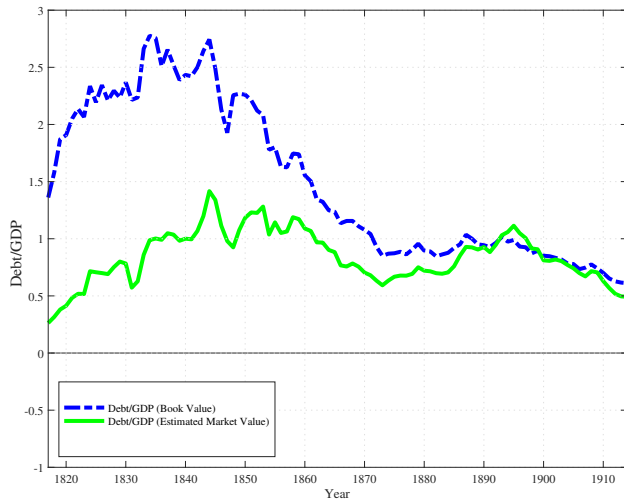
The Market Value of Outstanding UK Debt to GDP



The Book Value of Dutch Outstanding Debt to GDP



Market Value of Dutch Debt /GDP

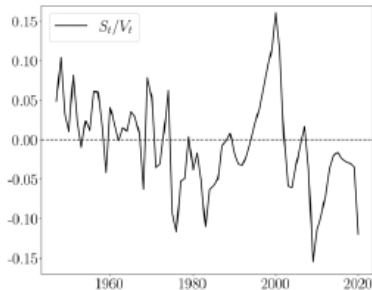


Key CGM (2023) Findings

1. **Traditional valuation ratio:** The market value of debt/output ratio has **unit root**.
 - ▶ Debt/output ratio in the U.S. is highly persistent. ✓
 - ▶ Unit root in market value of debt/output hard to reconcile with historical evidence; more consistent with slow mean-reversion in debt/output.
 - ▶ Market values can be inflated using financial repression (see [Hall, and Sargent \(2019, 2022\)](#) on WW-I, Interbellum, and WW-II)
 - ▶ Unit root hard to reconcile with underlying economics;
 - ▶ Hard to write down fully specified model that produces a unit root in market value of debt/output ratios (even with rational bubbles).
2. **New valuation ratio:** The surplus/debt ratio is stationary (the CGM (2023) predictor is the surplus/debt ratio)

Key CGM (2023) Findings

1. Traditional valuation ratio: The market value of debt/output ratio has **unit root**
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Key CGM (2023) Findings

1. Traditional valuation ratio: The market value of debt/output ratio has **unit root**.
2. **New valuation ratio**: The surplus/debt ratio is stationary (the CGM predictor is the surplus/debt ratio)
 - ▶ Surplus/debt ratio in the U.S. is not as persistent. ✓
 - ▶ What drives **mean reversion** in surplus/debt ratio?
 - ▶ **In case of surpluses**: Low surplus/debt ratio is pushed back up by higher surpluses or lower returns ✓
 - ▶ **In case of deficits**: Low surplus/debt ratio is pushed back up by higher surpluses (smaller deficits) or higher returns ✗
 - ▶ **In case of deficits**: More debt pushes the low surplus/debt ratio up (higher surplus/debt ratio means cheaper debt only in case of surpluses)
 - ▶ U.S. has been running primary deficits for over 6 decades.
 - ▶ What to make of mean reversion in surplus/debt ratio when government is running deficits? Not clear we "want" mean reversion in this ratio.

Loglinear Approximation of Surplus/Debt Ratio

- ▶ new CGM (2023) predictor is sv_t ; CGM (2023) approximate $\log(1 + \frac{S_t}{V_t})$ as follows:

$$sv_t = k + \frac{1-\rho}{1-\beta} (\tau v_t - \beta x v_t),$$

where $\tau v_t = \log(\frac{T_t}{V_t})$ and $x v_t = \log(\frac{X_t}{V_t})$, and $\mathbb{E} \log(1 + \frac{S_t}{V_t}) = -\log \rho$.

- ▶ simplifies to:

$$sv_t = k + \frac{1-\rho}{1-\beta} (\tau_t - \beta x_t) - (1-\rho)v_t, \text{ where } \tau_t = \log T_t, x_t = \log X_t, \text{ and } v_t = \log V_t.$$

- ▶ sv_t always \searrow when $v_t \nearrow$ as $\frac{\partial sv_t}{\partial v_t} = -(1-\rho) < 0$ when $\rho < 1$
- ▶ US has been running zero primary surpluses over 8 decades: $\rho \rightarrow 1$ as $\mathbb{E} \log(1 + \frac{S_t}{V_t}) \rightarrow 0$
 - ▶ as $\rho \rightarrow 1$, we know that $\beta \rightarrow 1$: $sv_t/(1-\rho)$ does not depend on v_t ?

Loglinear Approximation of Surplus/Debt Ratio

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- ▶ sv_t always \searrow when $v_t \nearrow$ as $\frac{\partial sv_t}{\partial v_t} = -(1-\rho) < 0$ when $\rho < 1$
- ▶ US has been running negative primary surpluses over 6 decades: $\rho > 1$ as $\mathbb{E} \log(1 + \frac{S_t}{V_t}) \rightarrow < < 0$.
- ▶ sv_t always \nearrow when $v_t \nearrow$ as $\frac{\partial sv_t}{\partial v_t} = -(1-\rho) > 0$ when $\rho > 1$.

Forecasting

- CGM (2023) approximate $\log(1 + \frac{S_t}{V_t})$ as follows:

$$\frac{sv_t}{1-\rho} = \frac{k}{1-\rho} + \frac{1}{1-\beta} (\tau_t - \beta x_t) - v_t$$

where $\tau v_t = \log(\frac{T_t}{V_t})$ and $xv_t = \log(\frac{X_t}{V_t})$, and $\mathbb{E} \log(1 + \frac{S_t}{V_t}) = -\log \rho$.

- By iterating this forward T times and taking expectations, we obtain the debt valuation equation:

$$\frac{sv_t}{1-\rho} = \mathbb{E}_t \sum_{j=1}^T \rho^{j-1} \left(r_{t+j} - \frac{1}{1-\beta} \Delta \tau_{t+j} + \frac{\beta}{1-\beta} \Delta x_{t+j} \right) + \mathbb{E}_t \frac{1}{1-\rho} \rho^T sv_{t+T}.$$

- as $\rho \rightarrow 1$, we know that $\beta \rightarrow 1$: all the forecasting work is done by $(\tau_t - \beta x_t)$, not v_t (how much do **valuations** matter for the CGM (2023) findings?)

Bottomline CGM (2023): One of the Dogs Did Bark

- ▶ Surplus/debt ratio is a valuation ratio (akin to a yield on the entire government debt portfolio)
- ▶ What drives a low surplus/debt ratio back up?
 1. **Discount rates:** low surplus/debt ratio predicts lower future real returns ✗
 2. **Cash flows:** low surplus/debt ratio (or low surplus?) predicts higher future tax revenue growth (not lower spending growth) ✓
 3. **Nothing at all:** low surplus/debt ratio predicts predicts low future surplus/debt ratio ✗

$$\frac{sv_t}{1-\rho} = \mathbb{E}_t \sum_{j=1}^T \rho^{j-1} \left(r_{t+j} - \frac{1}{1-\beta} \Delta \tau_{t+j} + \frac{\beta}{1-\beta} \Delta x_{t+j} \right) + \mathbb{E}_t \frac{1}{1-\rho} \rho^T sv_{t+T}.$$

Suggestion: Univariate Implementation

- Estimate a system of univariate forecasting regressions for $\sum_{j=1}^T \rho^j \Delta \tau_{t+j}$, $\sum_{j=1}^T \rho^j \Delta x_{t+j}$, $\sum_{j=1}^T \rho^j r_{t+j}$, v_{t+j} using the lagged surplus/debt ratio as a predictor:

$$\sum_{j=1}^T \rho^j \Delta \tau_{t+j} = a_\tau + b_T^\tau s v_t + \epsilon_{t+T}^\tau,$$

$$\sum_{j=1}^T \rho^j \Delta x_{t+j} = a_x + b_T^x s v_t + \epsilon_{t+T}^x,$$

$$\sum_{j=1}^T \rho^j r_{t+j} = a_r + b_T^r s v_t + \epsilon_{t+T}^r,$$

$$\rho^T s v_{t+T} = \phi_0 + \phi_T s v_t + \epsilon_{t+T}^v.$$

- More reliable estimates of long-run dynamics than VAR (Jordà, 2005)
- Cochrane (2008); Lettau and Van Nieuwerburgh (2008) adopt the same approach to implementing a Campbell-Shiller decomposition of the price/dividend ratio for stocks.

Take-away

- ▶ Agree with the authors that the US debt/output ratio is highly persistent.
- ▶ Skeptical about true unit root in market value of government debt/output.
- ▶ Would be good to have another valuation ratio for government debt that is less persistent.
- ▶ Surplus/Debt ratio is one candidate valuation ratio
- ▶ What does mean reversion in surplus/debt ratio mean when government is running deficits?

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