The Management of Talent: Optimal Contracting for Selection and Incentives*

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Abstract

Optimally reallocating human capital to tasks is key for an organization to successfully navigate a transition. We study how to design employment contracts to allocate employees to different valuable projects within an organization given two simultaneous challenges: (1) The employees have private information about their own cost of effort, and (2) they exert unobservable effort. The organization has two types of valuable projects, high and low impact, only the former of which requires unobservable effort. It would like to assign only an employee with low effort cost to the high-impact project. We characterize the optimal contract and show how it separates the employee types. The optimal contract menu pairs a higher probability of assignment to the high-impact project with a lower bonus in case of success. A fixed salary may also be used for the employees with high cost of effort, but only in limited cases. We link our results to job design features encountered in practice.

Keywords: optimal employment contracts, internal labor markets, adverse selection, moral hazard.

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1 Introduction

A defining challenge of organizations is the allocation of their staff to tasks. This is particularly the case when organizations undergo major transitions. For instance, public sector organizations face this challenge after political regime changes or administrative reorganizations. The same goes for private sector organizations following mergers, acquisitions, or abrupt changes in leadership. The better an organization can allocate its human capital, the better the chances that its transition will be a success.[1]

Allocating human capital after major transitions poses two specific challenges. First, during these circumstances, organizations are more likely to encounter asymmetric information along dimensions better known to the employee than to the leadership. Specifically, after regime changes, employees have private information about their own cost of working under the new regime. For instance, public sector bureaucrats who started their careers under one political regime may find it ideologically costly to do high-impact work that benefits the new political leadership. Similarly, employees of a small company acquired by a larger organization may not have the same motivation to work on innovative projects in the new organizational context. Second, employees must be motivated to exert effort in their new roles. Political regime changes often lead to different promotion criteria that reflect the priorities of the new administration, and an acquisition similarly leads to changes to the employees' bonuses. These two challenges map to the familiar problems of adverse selection and moral hazard.

We develop a model to analyze how organizations should optimally assign employees to projects and compensate them in the presence of the two challenges outlined above.[2] In many settings with simultaneous adverse selection and moral hazard, the optimal contract does not separate the employee types: the organization must offer the same contract to

[1]For instance, evidence from private sector acquisitions shows that a firm's ability to reassign its acquired workers to new roles is crucial for an acquisition's success (Tate and Yang, 2016; Lee et al., 2018).

[2]Other literature (Van Wesep, 2010) studied contracting when the firm has more information than a potential employee about how well the two will match. We address the opposite case, where there is private information about the employee's fit, along dimensions better known to the employee.

all employees.[3] A main feature of these models is the organization's focus on achieving selection on the hiring margin. We show that this result is upended if we focus on employee allocation *within* the organization rather than *to* the organization. This distinction matters, because in the former case, the organization has multiple distinct, valuable projects to which it can assign its employees. In the latter case, the focus is on employing a worker for one particular project. When multiple valuable projects are available, the optimal contract can separate the employee types, and we show how this separation can be achieved. The resulting optimal contract possesses three economically meaningful features: employees split their time between high- and low-impact projects; bonuses are paid for success; and sometimes, a salary is paid independent of project or success.

In our model, a risk-neutral organization runs two types of projects: low and high impact. The low-impact project produces a fixed value and does not require effort (e.g., administrative work). The high-impact project produces a higher value, but its success depends on unobserved employee effort (e.g., R&D work, which requires effort towards innovation). The organization faces a risk-neutral employee who may have either high or low congruence with the organization. A high-congruence employee has a lower cost of exerting effort, and hence, is more valuable to the organization. The organization would like to assign a high-congruence employee to the high-impact project, with incentives for him to exert effort. It would like to assign a low-congruence employee to the low-impact project. However, congruence and effort are known only to the employee.

The organization may offer contracts of the most general form. First, it can specify a probability of assignment to the high-impact project. This setting is equivalent to specifying the fraction of time spent working on the high-impact project versus the low-impact project. Second, the organization can offer a bonus if the high-impact project is successful. Third, the organization may offer a fixed salary, independent of project type or outcome. Throughout, the employee has limited liability, so the organization cannot impose penalties.

Analyzing the optimal contract rests on the following key insight. In an environment with

[3] See Gottlieb and Moreira (2014).

both adverse selection and moral hazard, the organization cannot solve both those problems simultaneously. The optimal contract then reflects a balance between solving the selection problem and the incentives problem. This balance is determined by two decisions: how the employee trades off selection versus incentives when choosing a contract from a menu, and how the organization trades off these two elements when building a contract menu.

To ensure that not all employee types choose the same contract, the optimal contract must involve a trade-off for the employee. Otherwise, selection cannot be achieved. A high-congruence employee should resolve his trade-off in favor of one type of contract, while the low-congruence employee should prefer a different one. This trade-off is reflected in a contract that pairs a higher probability of assignment to the high-impact project together with a lower bonus for success. We show that if it does not create such a trade-off in the contract menu, the organization would have to pay the low-congruence employee a sub-optimally high rent for him to reveal his type. The employee's trade-off is therefore his marginal rate of substitution of bonus for probability of assignment–that is, how an increase in the bonus is valued relative to an increase in the probability of assignment. For instance, if the high-congruence employee has a higher marginal rate of substitution than the low-congruence employee, then the former exhibits a preference for assignment over bonus. This employee prefers a contract that resolves the selection versus incentives trade-off in favor of selection.

When designing its jobs, the organization trades off a higher likelihood of assigning the employee to the high-impact project versus offering him higher monetary compensation. This trade-off is captured by the organization's marginal rate of substitution between bonus and probability of assignment. If this marginal rate of substitution is higher when the employee has high congruence, then the organization prefers to resolve its trade-off in favor of assigning this employee more often to the high-impact project (and vice versa).

The organization's preferred resolution of its trade-off is easily mapped into an employment contract when the high-congruence employee also reaches the same trade-off resolution. That is, if both the organization and the high-congruence employee have a relative preference for assignment, then this employee is assigned to the high-impact project with higher probability. If the high-congruence employee solves his trade-off differently than the organization, then the high-congruence employee values relatively more the aspect of the contract that is relatively less valued by the organization. Designing a contract that is acceptable to the employee becomes costlier for the organization. It can still design jobs to reflect its preferred trade-off between selection and incentives; however, it must add a fixed salary to the low-congruence employee's contract. This fixed salary, offered instead of additional bonus, is more valuable for the low-congruence employee, because this type has a higher cost of effort. Thus, the fixed salary is an agency cost arising endogenously due to adverse selection in the internal labor market.

In Section 5, we discuss the results of our model in the context of two main applications: employment contracts in government agencies and in technology companies. We map the job design implications of our model to contract features encountered in these cases. Take, for instance, the trade-off between bonus and the probability of assignment. Government agencies run high-impact projects that involve both full-time staff and part-time contractors, with the latter receiving higher pay. Technology companies often allow engineers to allocate a fraction of their time to innovative projects.[4] When such projects are successful, employees may be granted promotions to lead teams working on that new product. This reward can result in higher bonuses for part-time innovators compared to full-time researchers. Although the primary intent of such job designs may not be self-selection of employee types, our results show that they may have the consequence of increasing the positive selection of employees.

Another job design implication we map to observed contracts in our applications is the use of a fixed salary. Monetary compensation independent of project assignment or output is commonly used in practice in the form of base pay or sinecures. Our model highlights a positive selection role for these arrangements, albeit only in limited cases.

We diverge from the broader literature on optimal contracting under simultaneous adverse selection and moral hazard (Sappington and Lewis, 2000; Bernardo et al., 2001, 2008; Inderst

[4] Google's "20 percent rule" is a well-known example.

and Klein, 2007; Gottlieb and Moreira, 2014; Chade and Swinkels, 2016) by considering an environment with two valuable projects, only one of which requires unobservable effort. In a related theoretical paper, Gottlieb and Moreira (2014) also study the optimal contract under simultaneous adverse selection and moral hazard, but when the organization only has one type of project. They show that it is optimal to offer all employees the same contract and to only pay them contingent on the success of the project. Our model extends their setup by allowing for a second type of project, one which provides some value to the organization but does not require unobservable effort. Once this additional project is available, the optimal contract may, in fact, separate the employee types and include a fixed salary for the low-congruence employee. Separating the employee types is optimal because the organization can capture some value from the low-impact project, and this more than compensates for any losses from not assigning the low-congruence type to the high-impact project. Relatedly, Sappington and Lewis (2000) also consider optimal contracts under adverse selection and moral hazard, when the organization only has one project, and they also obtain the result that not separating the employee types is optimal. Chade and Swinkels (2016) provide sufficient conditions under which the moral hazard problem can be decoupled from the adverse selection problem in the optimal contract. Our multi-project setting, however, does not satisfy those sufficient conditions, as the problem is multi-dimensional: the organization sets bonuses, assignment probabilities, and the fixed salary.

Our model is closely related to Jeon and Laffont (1999), who study the efficient mechanism for downsizing in the public sector. In their model, employees have heterogeneous productivities, but their effort is fixed; that is, there is adverse selection but no incentives problem. We extend their original question to organizations where incentive schemes can be modified. These include most private sector organizations, as well as public bureaucracies during major regime transitions.[5] A main result of their model also emerges in ours: probabilistic assignment is necessary for self-selection. Our richer model, however, shows how the incentives problem changes the other main results of their paper. First, a fixed mone-

[5]For case studies of reforms to incentives in the public sector, see Nunberg et al. (1999) and Gualmini (2008).

tary transfer is no longer key to separating the employee types, because using the trade-off between bonus and assignment probability is preferable. The fixed salary is used only in a special case, when the organization and the employee have different preferences for how to resolve the trade-off of selection versus incentives. This has the applied implication that severance pay is optimal only in a small set of cases. Second, we obtain the key insight that the optimal contract displays a trade-off between over-assignment to the high-impact project and under-incentivizing effort. The organization must select the low-type too often if it wants to offer high-powered incentives to the high-type.[6]

The results highlight a key case when employee self-selection is preferable: differential employee congruence with the organization. While this is different from assuming intrinsic motivation on the part of the high-congruence employee (Murdock, 2002; Benabou and Tirole, 2003; Besley and Ghatak, 2005; Delfgaauw and Dur, 2007), it leads to a related intuition: the high-congruence type exerts more effort for the same pay, which makes his selection preferable.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 provides three key benchmarks. Section 4 derives the optimal employment contract, and Section 5 discusses the model's implications in the context of two applications. Section 6 extends the model to multiple employee types and to multiple project outcomes. Section 7 concludes, and the Appendix contains the proofs.

2 The Model

We consider an environment with two risk-neutral players: a firm and an employee. The employee has one of two possible types, $\theta \in \{H, L\}$. This type is the employee's private information, and the firm has prior belief μ_θ that the employee is of type θ. The employee

[6]The insight that the low-ability employee may be assigned to the high-impact project with positive probability relates our paper to work on constraints to the efficient use of promotions (Ke et al., 2018) or the inefficient use of work requirements in conjunction with promotions (Barlevy and Neal, 2019). In a different setting (with dynamic moral hazard), Axelson and Bond (2015) also show how two contracts with different rewards and different probabilities of working on the high-impact task co-exist in equilibrium.

must be assigned to work on a project. This project can be either high-impact, the outcome of which depends on the employee's private effort, or low-impact, which can be perfectly monitored by the firm and produces a safe output with value $W > 0$ for the firm.

The high-impact project produces a verifiable outcome $y \in \{0, 1\}$, where $y = 1$ denotes a success, and $y = 0$ denotes a failure. The probability of a success, $q(e)$, depends linearly on the employee's effort $e : q(e) = q_0 + q_1 \cdot e$, with $q_0, q_1 \in \mathbb{R}$, $q_0 \in [0, 1), q_1 \geq 0$. The θ-type employee has cost of effort $c(\theta, e)$ for $e \in \mathbb{R}_+$. The cost function $c(\theta, e)$ is continuously differentiable with respect to effort $e \in [0, (1 - q_0)/q_1]$, $\lim_{e \to 0} c_e(L, e) = 0$, $\lim_{e \to (1-q_0)/q_1} c_e(H, e) \geq q_1$, and

$$c_e, c_{ee}, c_{eee} \geq 0.^{7} \tag{1}$$

Moreover, $c(H, e) \leq c(L, e)$, and the H-type has lower marginal cost of effort than the L-type: $c_e(H, e) < c_e(L, e)$ for each $e \in \mathbb{R}_+$.

We allow the firm to offer the most general form of contracts. By the revelation principle, we can focus on the contract such that the employee declares his type, and the firm offers an employment contract $\mathcal{C}_{\hat{\theta}}$ that depends on the employee's declared type $\hat{\theta}$. We can focus on the contracts in which the employee declares his true type, so $\hat{\theta} = \theta$ in equilibrium. Since the firm and employee are both risk-neutral, it is without loss of generality to focus on the following contract: a probability p_θ of being assigned to a high-impact project, a bonus b_θ paid out only in case of project success ($y = 1$), and a rent r_θ that is the fixed salary received by the employee, non-contingent on project assignment or outcome.[8] We assume that the employee has *limited liability*: $b_\theta \geq 0$ and $r_\theta \geq 0 \ \forall \theta, y$.[9]

Timing of Actions. The above setup can be summed up in the following description of the game:

[7] The subscripts e denote derivatives with respect to effort. Note also that, as long as q is concave, we can always re-measure effort such that linearity of $q(e)$ is obtained for a convex cost of effort $c(\theta, e)$.

[8] Since the employee is risk-neutral, his utility is quasi-linear in the monetary transfer and in the cost of effort. If $\widetilde{b}_\theta(y) \geq 0$ is the bonus after y and $\widetilde{r}_\theta \geq 0$ is the fixed salary, re-define $b_\theta = \widetilde{b}_\theta(1) - \widetilde{b}_\theta(0)$ and $r_\theta = \widetilde{r}_\theta + \widetilde{b}_\theta(0)$. It is then without loss to assume $\widetilde{b}_\theta(1) \geq \widetilde{b}_\theta(0)$, since otherwise, the effort would be zero (and $\widetilde{b}_\theta(1) = \widetilde{b}_\theta(0) = 0$ would be optimal).

[9] Otherwise, "selling a project" to the employee, with a price accepted only by the H-type, is optimal.

1. The employee privately observes his type θ and declares his type.

2. The employee is paid a fixed salary (a rent) r_θ.

3a. With probability p_θ, the firm assigns the employee to the high-impact project and announces a bonus b_θ. After assignment, the employee chooses private effort $e(\theta, b_\theta)$. The firm receives value $y \in \{0, 1\}$ from the project, and it pays the bonus if $y = 1$.

3b. With probability $1 - p_\theta$, the firm assigns the employee to the low-impact project. The firm obtains a fixed value W, and the employee receives no additional bonus.

The Employee's Problem

If the θ-type employee is assigned the high-impact project with bonus b, his optimal effort choice is

$$e\,(\theta, b) = \arg\max_e q(e) \cdot b - c\,(\theta, e)\,. \tag{2}$$

Given the properties of the cost function $c\,(\theta, e)$, it follows that, for any bonus b, the H-type employee exerts higher effort than the L-type employee, $e\,(H, b) > e\,(L, b)$. Moreover, the maximum effort exerted satisfies $e\,(H, b_H) < 1$ with the equilibrium contract b_H, since it is suboptimal for the firm to offer $b_\theta \geq 1$. Given the employee's optimal choice of effort, his expected payoff after assignment to the high-impact project with bonus b is

$$V\,(\theta, b) \equiv q(e\,(\theta, b)) \cdot b - c\,(\theta, e\,(\theta, b))\,. \tag{3}$$

Then, this employee's ex-ante expected payoff from the employment contract (p, b, r) is given by

$$p \cdot V\,(\theta, b) + r. \tag{4}$$

The Firm's Problem

Assigning a θ-type employee the high-impact project yields a successful product with probability $q\left(e\left(\theta, b_\theta\right)\right)$, and it generates a cost of effort $c\left(\theta, e\left(\theta, b_\theta\right)\right)$ for the employee. Thus, the expected social value of a θ-type employee working on the high-impact project is

$$S\left(\theta, b_\theta\right) \equiv q\left(e\left(\theta, b_\theta\right)\right) - c\left(\theta, e\left(\theta, b_\theta\right)\right).$$

Out of this value, an expected payoff $V\left(\theta, b_\theta\right)$ is extracted by the employee, leaving the firm with the following expected profit from the high-impact project:

$$\pi\left(\theta, b_\theta\right) \equiv S\left(\theta, b_\theta\right) - V\left(\theta, b_\theta\right). \tag{5}$$

We make the following assumption about this profit relative to the profit from the low-impact project.

Assumption 1 *Under complete information about the employee's type, the firm would assign the L-type to the low-impact project and the H-type to the high-impact project. That is, the following inequality is satisfied:*

$$\max_{b_L} \pi\left(L, b_L\right) < W < \max_{b_H} \pi(H, b_H).$$

Assumption 1 states that the firm's payoff from running the low-impact project, W, is higher than its expected profit from running the high-impact project with a low-type employee; however, it is lower than its expected profit from running the high-impact project with a high-type employee. We are therefore in a setting in which the firm would like to only select the congruent employee (type H) for the high-impact project.[10]

The problem for the firm is to design a menu of contracts $\{\mathcal{C}_H, \mathcal{C}_L\}$ in order to maximize

[10] We relax this assumption in Online Appendix G.

its expected profit and to induce truthful revelation by the employee. Its expected profit is

$$\max_{\{p_H,p_L,b_H,b_L,r_H,r_L\}} \mu_H \cdot [p_H \cdot \pi(H, b_H) + (1-p_H) \cdot W - r_H] + \mu_L \cdot [p_L \cdot \pi(L, b_L) + (1-p_L) \cdot W - r_L], \quad (6)$$

subject to the incentive compatibility constraints

$$p_H \cdot V(H, b_H) + r_H \geq p_L \cdot V(H, b_L) + r_L, \quad (IC_H)$$

$$p_L \cdot V(L, b_L) + r_L \geq p_H \cdot V(L, b_H) + r_H. \quad (IC_L)$$

Constraint (IC_H) ensures that the H-type employee prefers the contract $\mathcal{C}_H = (p_H, b_H, r_H)$ to the contract designed for the L-type employee, $\mathcal{C}_L = (p_L, b_L, r_L)$. Similarly, constraint (IC_L) ensures that the L-type employee prefers the contract $\mathcal{C}_L$ to the contract designed for the H-type employee. The employee's participation is guaranteed by the limited liability assumption and the zero outside options. Throughout the paper, we analyze the non-trivial case in which the firm assigns at least one employee type to the high-impact project with positive probability, so $\max\{p_H, p_L\} > 0$.

3 Benchmarks

In order to articulate the role of simultaneous adverse selection and moral hazard, it will be useful to compare optimal contracts under our main model to those that arise in three benchmarks: complete information, verifiable types only, and verifiable effort only.

Consider first the case where employee type and effort are verifiable. This case delivers the first-best contract, which maximizes the joint surplus for firm and employee. For the low-impact project, this equals W, while for the high-impact project, this is equivalent to its social value given $b = 1$, $S(\theta, 1)$.

Proposition 1 *(Full-Information) With verifiable types and effort levels, the optimal con-*

tract for the firm has the following properties:

1. *The θ-type employee, when assigned to the high-impact project, exerts effort e_θ^{FB}, which generates social welfare $S(\theta, 1)$:*

$$e_\theta^{FB} \equiv \arg\max_e \left(q\left(e\right) - c\left(\theta, e\right)\right).$$

2. *The H-type employee is assigned to the high-impact project with probability one.*

3. *The L-type employee is assigned to the high-impact project with probability one if $S(L, 1) \geq W$; otherwise, he is assigned to the low-impact project with probability one.*

If an employee is assigned to the high-impact project, he exerts the socially efficient effort level for his type. By Assumption 1, an H-type employee working on the high-impact project generates more profit than the value of the low-impact project. This implies that the joint surplus from the high-impact project is also higher than W (given the employee's limited liability). The L-type employee working on the high-impact project generates less profit than the value of the low-impact project. The total surplus from the high-impact project could still be higher than W, if the employee captures high rents due to unobservability of effort.

Proposition 2 *(Incentives Provision Only) With verifiable employee type, in the optimal contract, the H-type employee is assigned to the high-impact project and offered an output-contingent bonus ($p_H = 1$, $b_H > 0, r_H = 0$), and the L-type employee is assigned to the low-impact project and given zero compensation ($p_L = 0$, $b_L = r_L = 0$).*

If the firm could observe employee types, it would assign the H-type to the high-impact project and offer him high-powered incentives. The firm would assign the L-type to the low-impact project and, as this project requires no effort, the firm would not provide any incentives for effort. Finally, no fixed salary would be paid, since such a payment would simply be a loss to the firm.

Proposition 3 *(Selection Only) With verifiable effort, in the optimal contract, the H-type employee is assigned only to the high-impact project ($p_H = 1$), while the L-type employee may be assigned to the low-impact project with positive probability ($p_L \leq 1$). The H-type exerts the socially optimal effort, while the L-type's effort is distorted whenever he is assigned to the high-impact project with positive probability ($p_L > 0$). A fixed salary is not offered: $r_H = r_L = 0$.*

This benchmark reflects the "no distortion at the top" result in adverse selection models: the optimal contract for the H-type employee is not distorted. The L-type employee has a higher cost of effort, so he would find it too costly to mimic the H-type. Yet, the contract for the L-type must be distorted in order to make this contract unattractive for the H-type. The distortion is necessary only when the L-type is also assigned to the high-impact project. Only that project offers a bonus that is relatively more valuable to the H-type than the L-type. The fixed salary is not optimal: here, only one incentive compatibility constraint is binding, so only one instrument (high-powered incentives) is sufficient to separate the types.

In all but the third benchmark, the firm assigns each employee type to a project with probability one. In all cases, the firm never pays a fixed salary. In the next section, we show how these results change when we combine the need for incentives provision and selection, in an environment with two valuable project types.

4 Analysis of the Full Model

4.1 Preliminary Properties

We start by deriving some preliminary properties of the optimal contract.

Lemma 1 *The optimal contract has the following properties:*

1. *The L-type's incentive compatibility constraint (IC_L) holds with equality;*
2. *If $p_L > 0$ and $r_L > 0$, then the H-type's incentive compatibility constraint (IC_H) also holds with equality;*

3. No fixed salary is offered to the H-type employee ($r_H = 0$);

4. One employee type is assigned to the high-impact project with probability one (either $p_L = 1$ or $p_H = 1$).

First, the optimal contract ensures that the L-type is indifferent between his contract and the contract intended for the H-type. Given Assumption 1, the firm would prefer to assign the L-type to the low-impact project, which requires no unobservable effort, and to not pay him any rent. Constrained by the need to induce truthful reporting, the firm offers this type the minimum rent r_L or probability of assignment p_L necessary to make him indifferent between the two contracts. Relatedly, the second insight is that the firm would not find it optimal to use both a fixed salary and a positive probability of assignment as instruments to satisfy only one incentive compatibility constraint. It would choose the cheaper of these two instruments. For instance, the firm would benefit from substituting any positive salary r_L for an increased bonus b_L (as long as $p_L > 0$).[11] Such a substitution is, however, not possible if the high-powered incentives make the resulting contract more attractive for the H-type, that is, if constraint (IC_H) binds.

Third, if the H-type were offered a fixed salary, consider the exercise of decreasing the salary r_H and increasing the bonus b_H. In this case, more high-powered incentives would make this contract less appealing to the L-type, because this type has a higher cost of effort. Thus, the firm only benefits from giving the H-type high-powered incentives.

Finally, the firm's expected value from the contract is positive: the firm benefits from assigning a high-impact project to the H-type, even if this comes at the cost of sometimes assigning the project to the L-type. Thus, the firm gains from "scaling up" the contract, that is, proportionally increasing the probabilities of assignment and the fixed salary, such that the H-type is assigned the project as often as possible: until $p_H = 1$, or until $p_L = 1$ and p_H is as large as possible while keeping the L-type's incentives for self-selection.

[11] Note that $b_L < 1$. Otherwise, $b_L = 1$ and $r_L > 0$ would imply that the firm's profit from assigning the L-type to the high-impact project is negative, and by incentive compatibility, it is also negative when it faces the H-type.

These properties lead to the following key insight for the shape of the optimal contract.

Proposition 4 *In the optimal contract, the employee type who is assigned with lower probability for the high-impact project receives a higher bonus in case of success.*

This result shows that the distortions due to simultaneous moral hazard and adverse selection necessarily lead to a trade-off between high-powered incentives provision and project assignment. If the firm didn't offer a fixed salary, then a contract that offers both a higher bonus and higher probability of assignment would be preferable to all employee types. Hence, a trade-off between bonus and probability of assignment is necessary in order to satisfy incentive compatibility. Interestingly, the trade-off exists even if the L-type is offered a fixed salary ($r_L > 0$). Consider a contract with both lower assignment probability and lower bonus for the L-type, to which a fixed salary is added. If this contract makes the L-type indifferent between his and the H-type's contract, this means that the fixed salary is just enough to compensate the L-type for his expected loss from being given a lower probability of assignment and a lower bonus. From the perspective of the H-type employee, the L-type's contract is then strictly less desirable. He achieves project success more often than the L-type, so both assignment and bonus carry higher expected value for him. The fixed salary carries the same value for both types, so it would not be enough to compensate the H-type for his lost value from the lower probability of assignment and the lower bonus. As shown above, though, both incentive compatibility constraints must be binding in the optimal contract. Thus, the fixed salary cannot solve the self-selection problem without providing inefficient incentives instead.

Finally, a key component of our model is the low-impact project, which creates positive value for the firm and does not require unobservable effort. To highlight the importance of this project for our results, we describe the optimal contract if this project had no value.

Lemma 2 *If the low-impact project had no value to the firm ($W = 0$), then the same contract would be offered to all employee types:* $p_L = p_H = 1$, $b_L = b_H > 0$, $r_H = r_L = 0$.

Without a valuable low-impact project, the optimal contract assigns both employees only to the high-impact project. Incentivizing the L-type to select out would come at a cost, without sufficient benefit to the firm. With both employee types assigned to the high-impact project with the same probability, the firm maximizes profits by offering the same high-powered incentives (as implied by Proposition 4). The existence of a valuable low-impact project, which does not require unobservable effort, will prove critical for our main result that the firm can achieve employee self-selection through the employment contact.

4.2 The Optimal Contract

In this section, we proceed to derive the optimal contract. We begin with the extreme case in which the L-type has a very high cost of effort, such that he generates less social value by working on the high-impact project than the social value of the low-impact project. In this case, we show that the firm induces the L-type to self-select into the low-impact project by paying him an information rent in the form of fixed salary. Next, we consider the case in which both types of employees may generate sufficiently high social value by working on the high-impact project. We will show that analyzing this case rests on understanding how both the firm and the employee trade off the bonus versus the probability of assignment to the high-impact project. Finally, we will characterize the optimal contract and show when a fixed salary is optimally added to the contract.

First, if the L-type is very unproductive on the high-impact project compared to the value of his work on the low-impact project, then the firm only assigns him to the latter:

Proposition 5 *If $W \geq S(L,1)$, then only the H-type employee is assigned to the high-impact project ($p_L = 0$, $p_H = 1$), and the L-type employee receives a fixed salary $r_L > 0$. Conversely, if $W < S(L,1)$, then the L-type is assigned to the high-impact project with positive probability ($p_L > 0$).*

When $W \geq S(L,1)$, the L-type employee creates less social value by working on the high-impact project than by working on the low-impact project. Then, it is preferable for the firm to assign this type to the project where he will create more value, and to offer him a fixed salary that makes his contract as appealing as taking the contract designed for the H-type. Conversely, when $W < S(L, 1)$, there is more social value in assigning the L-type employee to the high-impact project than to the low-impact project. The firm can capture some of this value by sometimes assigning the L-type employee to the high-impact project. This employee's expected surplus is determined by the binding incentive compatibility constraint, which ensures that he receives the same expected value as from the other type's contract. Thus, the firm maximizes the relative social surplus, and afterwards, it gives the employee the payoff dictated by the incentive constraints.

Corollary 1 *If $W \geq S(L, 1)$, then the H-type employee's effort e_H is lower than when the employee types are verifiable.*

Even when the L-type is assigned to the low-impact project with probability one, the H-type employee's effort is distorted compared to the case in which types are verifiable. This happens because the L-type's fixed salary is set so as to make this type indifferent between his and the H-type's contract. Thus, the firm distorts down the H-type's incentives for effort in order to reduce the L-type's rent.[12]

When $S(L, 1) > W$, Lemma 1 implies that either the optimal contract does not separate the employee types ($p_L = p_H = 1$), or one employee type is assigned to either project with probability less than one, while the other type is assigned to the high-impact project with probability one. Next, we show how the optimal contract is constructed, and when a differentiated contract is offered.

The Trade-off between Selection and Incentives

In what follows, we will shown when and why a contract that separates the employee types is preferred to a pooling contract by the firm. By Proposition 4, incentive compatibility

[12]The H-type employee's effort is also distorted compared to the full-information case. To see this, note that, by Proposition 1, the effort in the full-information case maximizes the social welfare. To achieve such effort, the firm would have to pay $b_H = 1$.

requires that the employee type who is assigned with lower probability to the high-impact project receives a higher bonus in case of success, that is, $p_\theta \leq p_{\theta'}$ if and only if $b_\theta \geq b_{\theta'}$ for each $\theta \neq \theta'$. We will show that the key to characterizing the optimal contract lies in understanding how this trade-off between bonus and assignment probability is resolved for both the firm and the employee.

To capture and analyze this trade-off, we find it illuminating to split the problem in two steps. First, we isolate the trade-off between bonus and assignment probability by restricting the analysis to the pairs of contracts that do not include a fixed salary: $\Psi^0 \equiv \{(p, b, 0) : p \in [0, 1], b \in [0, 1]\}$. Second, we will show when adding a fixed salary is necessary in order to improve on those contracts.

The Employee's Trade-off Consider the problem for the θ-type employee of selecting a contract when offered a choice between a pair of contracts from Ψ^0. The trade-off faced by the employee comes down to understanding how the employee values an incremental change in bonus pay relative to a decremental change in the probability of assignment. For the θ-type employee, this is captured by the marginal rate of substitution of bonus b for probability p, at a fixed expected payoff, $p \cdot V(\theta, b)$, that is, the employee's indifference curve traces[13]

$$\left|\frac{dp}{db}(\theta)\right| = p \cdot \frac{V_b(\theta, b)}{V(\theta, b)}. \tag{7}$$

The H-type has a lower marginal cost of effort than the L-type, so he obtains *both* higher marginal utility from an increase in the probability of assignment, $V(H, b) \geq V(L, b)$, and higher marginal utility from an increase in the bonus, $V_b(H, b) \geq V_b(L, b)$. Thus, the indifference curve may be steeper for either of the two employee types. We will say that *the employee indifference curves exhibit increasing preference for assignment* if the H-type's indifference curve is flatter than the L-type's for all contracts (p, b) in Ψ^0, that is,

$$\frac{V_b(H, b)}{V(H, b)} \leq \frac{V_b(L, b)}{V(L, b)}. \tag{8}$$

[13]For clarity of exposition, we use the absolute value of the slope of the indifference curve.

We say that *the employee indifference curves exhibit increasing preference for bonus* if the inequality goes in the other direction for all (p, b):

$$\frac{V_b\,(H, b)}{V\,(H, b)} \geq \frac{V_b\,(L, b)}{V\,(L, b)}. \tag{9}$$

Assumption 2 *The employee indifference curves either satisfy* (8) *for all contracts* (p, b) *in* Ψ^0*, or they satisfy* (9) *for all contracts* (p, b) *in* Ψ^0*.*

Under such monotonicity, we immediately obtain single-crossing of the indifference curves. This single-crossing property is illustrated in Figure 1. Each employee type's indifference curve slopes downward, as he trades off a lower bonus for a higher probability of assignment. Starting from an contract $\mathcal{C}_H$ offered to the H-type employee, we can find the indifference curve for the L-type that intersects the indifference curve for the H-type at $\mathcal{C}_H$. Then, the region of the parameter space in which an incentive-compatible contract $\mathcal{C}_L$ can be located relative to $\mathcal{C}_H$ is represented by the shaded area in Figure 1: the region above the L-type's indifference curve, where this type is better off than under contract $\mathcal{C}_H$, and below the H-type's indifference curve, where the H-type is worse off than under contract $\mathcal{C}_H$.

The Firm's Trade-off We now move to the firm's problem of selecting a contract menu from Ψ^0 to offer the employee. In this case, we see the social welfare as the firm's objective. To understand why considering the social welfare is useful, suppose that the firm offers a differentiated contract $(\mathcal{C}_H, \mathcal{C}_L)$ to the employee. By incentive compatibility, the H-type employee obtains a higher rent from $\mathcal{C}_H$ than from $\mathcal{C}_L$. If the social welfare from offering $\mathcal{C}_H$ to the H-type employee is not higher than offering him $\mathcal{C}_L$, then the firm would like to deviate and offer $\mathcal{C}_L$ to both types, since the firm's payoff is the social welfare minus the rent captured by the employee. Symmetrically, if the social welfare from offering $\mathcal{C}_L$ to the L-type employee is not higher than offering him $\mathcal{C}_H$, the firm would like to deviate and offer $\mathcal{C}_H$ to both types. In summary, it is a *necessary condition* for the firm's optimality that the social welfare from contract $\mathcal{C}_\theta$ is higher than the social welfare from contract $\mathcal{C}_{\theta'}$ (with

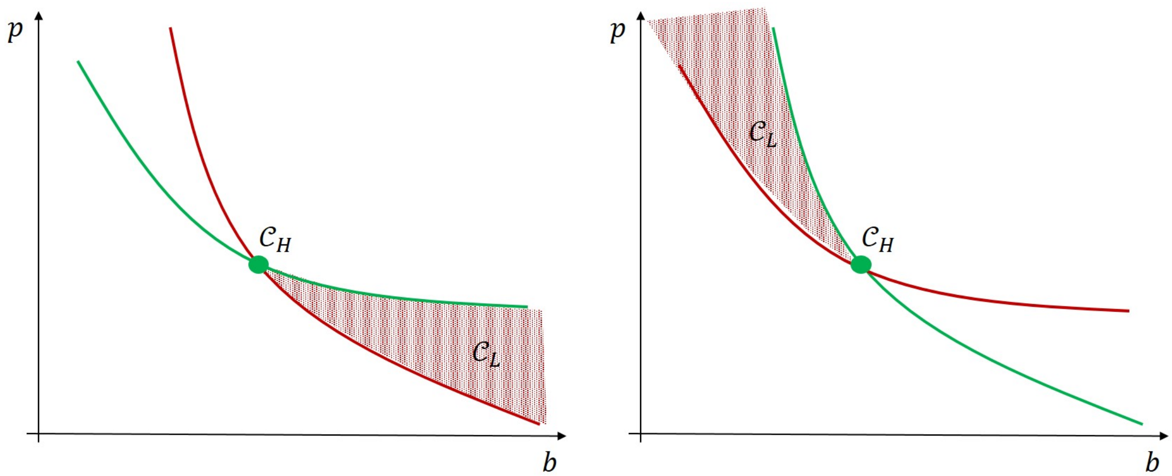


Figure 1: Depicts the employee indifference curves between bonus b and probability p, given a contract $\mathcal{C}_H$. Panel (a) depicts the case when the employee indifference curves exhibit increasing preference for assignment, and Panel (b) depicts the case when they exhibit increasing preference for bonus. The green curve is the H-type employee's indifference curve, and the red curve is the L-type employee's indifference curve. The two curves intersect at $\mathcal{C}_H$, the contract offered to the H-type. The red shaded area denotes the region where an incentive compatible contract $\mathcal{C}_L$ can be offered to the L-type. This is the region above the L-type's indifference curve and below the H-type's indifference curve.

$\theta' \neq \theta$) when the firm faces the type-θ employee.

As will be seen below, considering this necessary condition will provide us with a useful guidance towards finding the optimal contract. This is because, in equilibrium, the L-type employee's incentive compatibility constraint holds with equality. Hence, when the firm marginally changes the contract offered to the L-type, keeping the contract to the H-type fixed, the firm's marginal profit is identical to the marginal change in social welfare.[14]

Seeing the social welfare as the firm's objective, the trade-off for the firm when facing a θ-type employee is then

$$\left|\frac{dp}{db}(\theta)\right| = p \cdot \frac{S_b(\theta, b)}{S(\theta, b) - W}. \tag{10}$$

The H-type employee both exerts more absolute effort for the same bonus and exerts marginally more effort in response to an increase in the bonus. This means that the net

[14]One may wonder why we do not consider the firm's payoff $p \cdot \pi(\theta, b) + (1-p) \cdot W$ directly. The issue is that, since $\pi(\theta, b)$ is not monotone in b, we do not have a tractable regularity.

social welfare increases more when the H-type is given a higher probability of assignment, as opposed to the L-type, that is, the denominator of (10) is higher for the H-type. It also increases more when the H-type is given a higher bonus, that is, the numerator of (10) is also higher for the H-type. Thus, (10) may be higher when facing either type of employee. We will say that *the firm's indifference curves exhibit increasing preference for assignment* if (10) is higher when the firm is facing the H-type employee, for all contracts (p, b) in Ψ^0:

$$\frac{S_b(H, b)}{S(H, b) - W} \leq \frac{S_b(L, b)}{S(L, b) - W}, \tag{11}$$

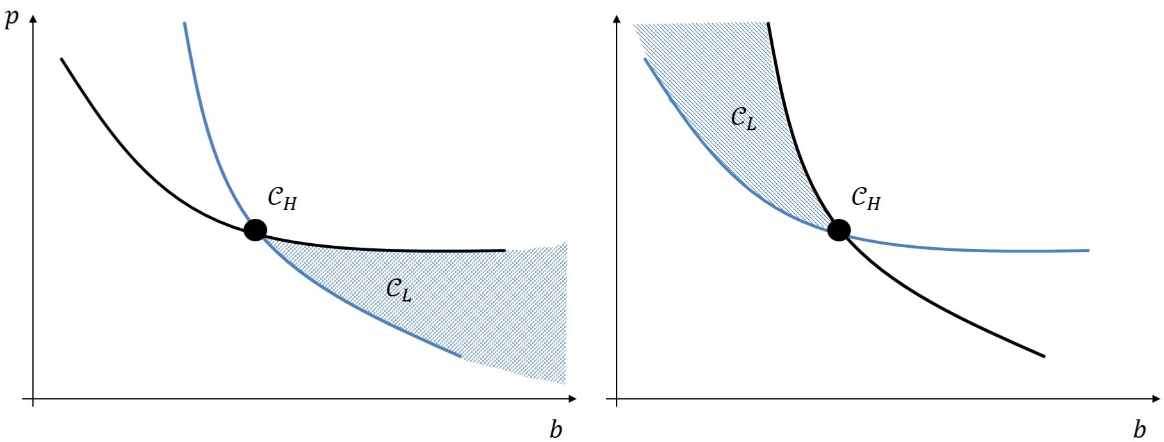
and that *the firm's indifference curves exhibit increasing preference for bonus* if the inequality goes in the other direction for all (p, b):

$$\frac{S_b(H, b)}{S(H, b) - W} \geq \frac{S_b(L, b)}{S(L, b) - W}, \tag{12}$$

Under such monotonicity, we immediately arrive at another single-crossing. The indifference curve for the firm facing the H-type employee and the indifference curve for the firm facing the L-type employee intersect only once.

This single-crossing property is illustrated in Figure 2. Each curve slopes downward, as the firm trades off a lower bonus for a higher probability of assignment. Consider starting from a contract $\mathcal{C}_H$ offered to the H-type employee. The shaded region in Figure 2 represents the area of the parameter space in which a contract $\mathcal{C}_L$ can be located relative to $\mathcal{C}_H$ in order to make the firm better off from separating the employee types. This is the region above the firm's indifference curve when facing the L-type and below the firm's indifference curve when facing the H-type.

In order to streamline the analysis and highlight the mechanism behind the main result, we present the results for the case in which *the employee indifference curves exhibit increasing preference for assignment*, and *the firm's indifference curves exhibit either increasing*



(a) Increasing Preference for Assignment

(b) Increasing Preference for Bonus

Figure 2: Depicts the firm's indifference curves between bonus b and probability p. Panel (a) depicts the case when the firm's indifference curves exhibit increasing preference for assignment, and Panel (b) depicts the case when they exhibit increasing preference for bonus. The black curve is the firm's indifference curve when facing an H-type employee, and the blue curve is the firm's indifference curve when facing the L-type employee. The two curves intersect at $\mathcal{C}_H$, the contract offered to the H-type. In the blue shaded area, offering a contract $\mathcal{C}_L$ to the L-type could make the firm better off (again, this is the necessary condition discussed above). If the type-L's incentive constraint holds with equality, then offering $\mathcal{C}_L$ is in fact profitable.

preference for assignment or increasing preference for bonus.[15]

Implications for the Optimal Contract

Consider the case where both the employee's and the firm's preferences are in accord, so that all indifference curves exhibit increasing preference for assignment.

Proposition 6 *Suppose both the employee and the firm indifference curves exhibit increasing preference for assignment, and $W < S(L, 1)$. Then the optimal contract has the following properties:*

1. *No fixed salary is offered ($r_L = r_H = 0$);*

2. *The H-type employee is assigned to the high-impact project ($p_H = 1$), and the L-type employee is assigned to either project with positive probability ($p_L \leq 1$, with $p_L < 1$ when the contract separates the employee types);*

[15] These conditions are satisfied under a broad set of cost functions $c(\theta, e)$, and examples are provided in the Online Appendix. In the Online Appendix F, we provide the analysis for the other possible cases.

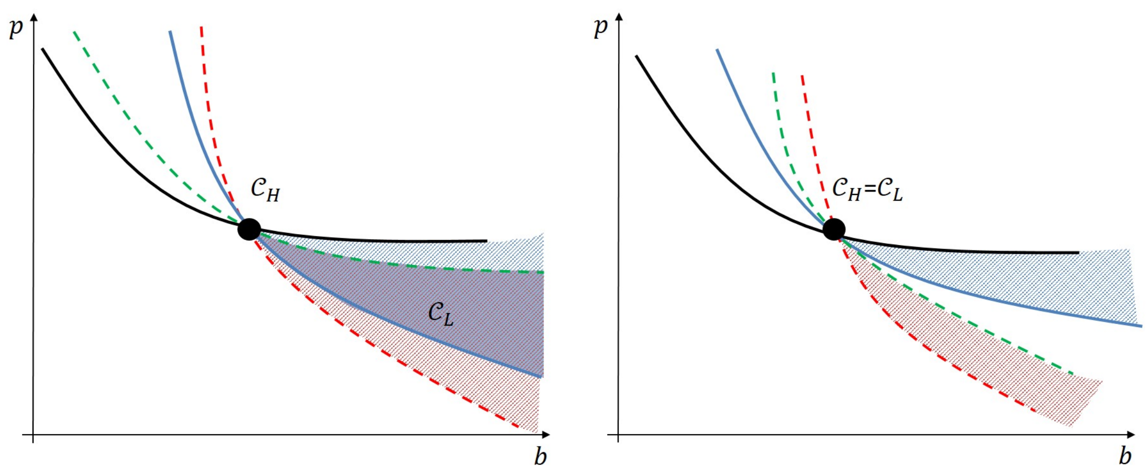
3. *The L-type employee is offered a higher bonus than the H-type ($b_L \geq b_H$).*

We discuss this result in the two steps described above: first, we describe the feasible set of contracts without the fixed salary; and second, we examine whether a fixed salary can improve on the first solution. Starting from the set of contracts without a fixed salary, consider the combined insights from Figures 1 and 2. Figure 3 combines the Panels (a) of these two figures in one graph. It illustrates the set of separating contracts that could increase the firm's profit (the blue shaded region) and that are incentive compatible for the employee (the red shaded region). As shown in the figure, when there is increasing preference for assignment on the side of both the employee and the firm, these regions may overlap. Thus, it may be possible for the firm to find a contract menu in Ψ^0 that improves on its profit and separates the employee types.[16] This happens because the firm and the H-type employee both value selection relatively more than incentives for effort, compared to the L-type employee. The firm would rather increase the probability of assigning the H-type to the high-impact project, and the H-type employee values the contract with higher p relatively more than does the L-type employee.

The next step is to ask whether adding a fixed salary can further improve the firm's profit. Consider starting from a contract without a fixed salary. The firm could modify this contract to add a fixed salary and decrease the bonus such that the employee's expected payoff is unchanged. This decrease in bonus is higher for the L-type employee, because this employee achieves success less often, so he is willing is give up more bonus for the same fixed salary. Thus, the firm can construct an incentive compatible contract menu where it gives the L-type employee a fixed salary, a lower bonus, and, given the insight from Propostion 4, a higher probability of assignment to the high-impact project.

Offering a contract where the H-type's probability of assignment is lower than that of the L-type would not, however, be profit-enhancing for the firm, as it values selection of the

[16]Up to this point, we only satisfy the necessary condition. Hence, it is possible that the firm cannot find a profit-improving separating contract and decides to offer the same contract to both types. See Corollary 2 for the full characterization of the optimal contract.



(a) Optimal Contract Separates Types (b) Optimal Contract is Same for Both Types

Figure 3: Depicts the employee and the firm indifference curves between bonus b and probability of assignment p, given a contract $\mathcal{C}_H$, when both firm and employee indifference curves exhibit increasing preference for assignment. The green curve is the H-type employee's indifference curve, and the red curve is the L-type employee's indifference curve. The black curve is the firm's indifference curve when it faces an H-type employee, and the blue curve is the firm's indifference curve when it faces a L-type employee. In the red shaded area, an incentive compatible contract $\mathcal{C}_L$ can be offered to the L-type. In the blue shaded area, offering a contract $\mathcal{C}_L$ could be profit-increasing for the firm. Panel (a) gives the case where there is overlap between the two shaded areas, so a separating contract could be profit-enhancing for the firm and incentive compatible. Panel (b) gives the case where there is no overlap between the two shaded areas, so the firm prefers not to offer a separating contract.

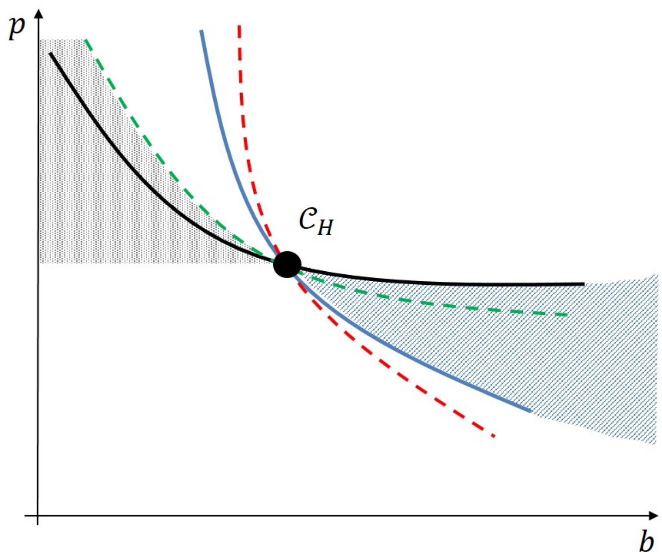


Figure 4: Depicts the case where both firm and employee indifference curves exhibit increasing preference for assignment. The green curve is the H-type employee's indifference curve, and the red curve is the L-type's indifference curve. The black curve is the firm's indifference curve when it faces an H-type employee, and the blue curve is the firm's indifference curve when it faces a L-type employee. In the blue shaded area, offering a contract $\mathcal{C}_L$ would be profit-increasing for the firm, compared to offering the same contract $\mathcal{C}_H$ to all employee types. Contracts in the gray shaded area are incentive compatible with a fixed salary.

H-type relatively more. Figure 4 illustrates this insight: the region in which the additional feasible contracts can be located, shaded in gray, does not overlap with the region in which we satisfy the necessary condition for the firm's profit to increase, shaded in blue.[17] Thus, the optimal contract does not offer a fixed salary. Rather, it reflects the firm's relative valuation of selection versus incentives for effort: it offers the H-type employee a higher probability of assignment to the high-impact project, but a lower bonus.

Next, we consider the case where there is dis-accord between the employee's and the firm's preferences over the trade-off between selection and incentives.

Proposition 7 *Suppose the employee indifference curves exhibit increasing preference for assignment, the firm's indifference curves exhibit increasing preference for bonus, and $W < S(L, 1)$. Then the optimal contract has the following properties:*

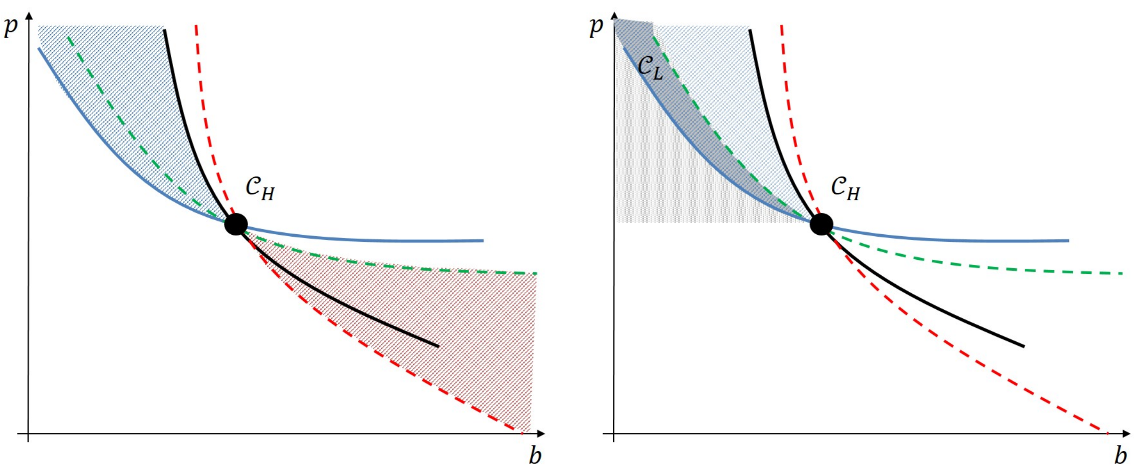
[17]Notice that adding the fixed salary would not change the firm's indifference curves, since the fixed salary is not part of the social welfare function, as it is only a transfer.

1. *A fixed salary for the L-type is part of any contract that separates the types ($r_L > 0$ if and only if $p_L \neq p_H$);*

2. *The L-type employee is assigned to the high-impact project ($p_L = 1$), and the H-type employee is assigned to either project with positive probability ($p_H \leq 1$, with $p_H < 1$ when types are separated);*

3. *The H-type employee is offered a higher bonus than the L-type ($b_H \geq b_L$).*

We again use our two-step analysis to gain insight into this result. First, we focus on the feasible set of contracts without the fixed salary. Figure 5 combines both the employee's and the firm's indifference curves for this case in one graph. The firm's indifference curves exhibit increasing preference for bonus, which implies that the firm values incentives for effort relatively more than selection when it is facing the H-type employee, compared to when it is facing the L-type employee. Thus, the set of contracts that maximize the firm's profit has to offer the H-type employee a lower probability of assignment to the high-impact project, but more high-powered incentives. Yet, as illustrated in Panel (a), the incentive compatible contracts when the employee indifference curves exhibit increasing preferences for assignment must offer the H-type a higher probability of assignment than the L-type. Thus, without a fixed salary, the firm cannot offer a separating contract that is more profitable than offering the same contract to all employee types. As discussed above, adding a fixed salary to the L-type's contract allows the firm to offer incentive compatible contracts where the H-type receives higher powered incentives. This is exactly the type of contract preferred by the firm for this employee type. Panel (b) of Figure 5 illustrates this insight: the region in which the additional feasible contracts can be located, shaded in gray, overlaps with the region in which the necessary condition for the firm's profit to increase holds, shaded in blue. The optimal contract either offers the same contract to both types or offers a fixed salary to the L-type employee. Also, the optimal contract reflects the firm's relative valuation of selection versus incentives for effort: it offers the H-type employee higher powered incentives and a lower probability of assignment to the high-impact project.



(a) Without A Fixed Salary (b) With a Fixed Salary

Figure 5: Depicts the case where the employee (firm's) indifference curves exhibit increasing preference for assignment (bonus). The green (red) curve is the H-type's (L-type's) indifference curve. The black (blue) curve is the firm's indifference curve when it faces an H-type (L-type) employee. Panel (a) illustrates the case where no fixed salary is offered. The red shaded area is the region where an incentive compatible contract $\mathcal{C}_L$ can be offered to the L-type. In the blue shaded area, offering a contract $\mathcal{C}_L$ could be profit-increasing for the firm. Panel (b) illustrates the case where a fixed salary is added to the contract. In the gray shaded area, an incentive compatible contract $\mathcal{C}_L$ can be offered to the L-type, with a fixed salary. The overlap between the gray and the blue regions is the area where the firm may be able to find a profit-increasing, incentive compatible contract $\mathcal{C}_L$ with a fixed salary.

The insight from the above results may be summarized as follows. In an environment with both moral hazard and adverse selection, the firm cannot solve both problems simultaneously. It faces a trade-off between solving adverse selection and solving moral hazard. In either case, it can separate the employee types by prioritizing the part of the contract that brings it the higher marginal benefit. How costly it is to achieve separation depends, however, on how the employee values one part of the contract relative to the other. When the high-type employee's relative valuation of bonus versus probability of assignment goes in the opposite direction to the firm's, separation is costlier. The firm pays the low-type employee a fixed salary. This salary is an agency cost that arises endogenously to compensate for the firm's, and the H-type employee's, different relative valuations of selection versus incentives.

Characterization of the Optimal Contract

We will now describe the contracting problem in the two cases we identified in the previous section. First, when both the employee and the firm indifference curves exhibit increasing preference for assignment, constraint (IC_L) binds and constraint (IC_H) does not bind. The H-type employee is assigned to the high-impact project, $p_H = 1$, and no fixed salary is offered, $r_H = r_L = 0$. Substituting these results into (6), we obtain

$$\max_{b_L \geq b_H \geq 0} \mu_H \cdot (\pi(H, b_H) - W) + \mu_L \cdot \frac{V(L, b_H)}{V(L, b_L)} \cdot (\pi(L, b_L) - W). \tag{13}$$

Corollary 2 *When both the employee and the firm indifference curves exhibit increasing preference for assignment, the optimal contract solves (13).*

Next, consider the case in which the employee indifference curves exhibit increasing preference for assignment and the firm indifference curves exhibit increasing preference for bonus. In this case, both (IC_H) and (IC_L) hold with equality (because the firm either offers the same contract to both types or offers a fixed salary to the L type), and substituting into (6), we obtain

$$\max_{b_H \geq b_L \geq 0} \mu_H \cdot \frac{V(H, b_L) - V(L, b_L)}{V(H, b_H) - V(L, b_H)} \cdot (\pi(H, b_H) - W) \\ + \mu_L \cdot (\pi(L, b_L) - W - V(L, b_H) + V(L, b_L)). \quad (14)$$

Corollary 3 *When the employee indifference curves exhibit increasing preference for assignment and the firm indifference curves exhibit increasing preference for bonus, the optimal contract solves (14). Moreover, a fixed salary is offered ($r_L > 0$) if and only if $b_H > b_L$.*

In the Online Appendix E, we provide examples for each of these two cases.

5 Applied Implications

Our model has several implications for the type of employment contracts and job designs created when both employee congruence and motivating effort are important concerns. These concerns are likely to emerge when organizations undergo major transitions, and they are likely heightened when firing employees is very costly, due to labor laws, union agreements, concerns about the effects on morale, or prohibitive costs of training new employees.

Below, we discuss our model's implications through the lens of two applications. Our model is highly stylized, and it abstracts from many other sources of friction present in these applications. Nevertheless, we discuss how our results speak to the effects of the job designs seen in practice in each case.

Government Agencies

Consider first the case of government agencies. The issue of ideological congruence between political leaders and bureaucrats employed in government agencies has been extensively studied in the political economy literature (see Gailmard and Patty (2012) for a survey). Congruence is an important issue when the bureaucrats run high-impact projects–creating policy, for example, or writing the content and application of regulations. In these cases, a bureaucrat who is not ideologically congruent with the political leadership will find it more costly to work on high-impact projects. Moreover, bureaucratic effort is generally not verifiable by the political leadership, as it involves expertise developed within the bureaucracy (Bendor and Meirowitz, 2004).

Congruence is not a concern for tasks where discretion is limited: for example, low-impact projects like procurement planning. Government agencies run a wide variety of projects, covering the range from high to low impact. We therefore have the key elements of our model. There is private information on the side of the employee, unobservable effort, and two types of projects, of which the high-impact ones require unobservable effort.

The job designs encountered in government agencies also allow for the three contract features necessary for employee self-selection. First, the range of projects ran by any agency allows for job designs that split employee time between high- and low-impact projects.[18] Second, the bonus for a project's success may come either in the form of pay-for-performance (OECD, 2003), or in the form of a promotion.[19] Third, the fixed salary exists in the form of base pay.

The job design implications of our model can be linked to contract implementations seen in practice. We discuss how these contract features help solve the joint self-selection and incentives problem, given the insights from our model. These effects are present regardless of whether or not these observed contract features were introduced specifically in order to solve the problems discussed in this paper.

Consider the case where the high-congruence public sector employee is the one who values assignment relatively more than bonus pay, and that the trade-off for the government agency's leadership goes in the same direction. This direction of the trade-off has an intuition similar to that present in the intrinsic-motivation literature, in that the high-congruence employee is the one who exerts more effort for the same compensation. Given the frequent link made between public employment and intrinsic-motivation (Besley and Ghatak, 2005;

[18]Probabilistic assignment in our model can be directly mapped to specifying the fraction of time the employee spends on each project. Details available upon request.

[19]Our model is static, but one can think of the bonus as the net present value of a promotion.

Gailmard and Patty, 2007; Delfgaauw and Dur, 2010; Perry et al., 2010), we contend that this direction of our trade-off is more suited for this case than the alternative.

Given the above conditions, Proposition 6 shows that the best contract assigns the high-congruence employee to the high-impact project more often than the low-congruence employee; however, it offers the high-congruence employee a lower bonus for success. Job designs with these features are observed in practice. For instance, consider the job designs for civil servants versus government contractors working for the same agency.[20] The civil servants are assigned the high-impact projects like development of regulations, and government contractors are sometimes assigned to support high-impact projects–for example, to support with the development of regulations–and sometimes they are assigned to projects classified as low-impact, such as support in procurement planning.[21] Nevertheless, the bonus for success is arguably higher for government contractors. First, pay-for-performance contracts are more common for government contractors than for civil servants (OECD, 2003). Second, a contractor's future access to work is dependent on the quality of their output, more so than in the case of a civil servant. Moreover, average pay data shows that the overall compensation of contractors tends to be higher.[22]

Given the insights of our model, the above job design has the benefit of selecting ideologically congruent employees to undertake high-impact projects in government agencies. Our results also imply that, in this context, other job designs do not satisfy the requirement that each employee does not prefer a contract designed for another type of employee. Thus, other job designs do not separate the employee types.

R&D in Private Companies

Next, consider the case of technology companies. They run both high-impact projects–for example, R&D, which requires unobservable effort towards innovation–and low-impact

[20] Most government agencies use contractors. According to federalpay.org, "contractor" was the 26th most common type of federal occupation in the United States in 2018.

[21] For instance, for a list of projects that may be assigned to government contractors in the Department of Energy, see https://www.energy.gov/management/contracting-support-services, accessed March 28, 2020.

[22] Data on average compensation is publicly available at federalpay.org, accessed March 26, 2020.

projects, such as coding an application. A major transition may be triggered by the acquisition of a smaller company by a larger organization. In the technology sector, it is not uncommon for such acquisitions to be made for the primary purpose of accessing the smaller company's human capital. Since 2009, in fact, there has been an empirically documented increase in the number of such acquisitions (Chatterji and Patro, 2014).[23] This process naturally triggers the need to reallocate human capital, as the smaller company's projects are discontinued.

In this context, we again encounter the key elements of our model. Private information on the side of the employee is a concern, because these acquired employees, many of them former entrepreneurs, may plausibly know better their desire to pursue innovative work within a large firm, or their fit with this firm's mission. Motivating these employees to exert effort matters, and the firm runs a gamut of projects, only some of which require unobservable effort, such as effort towards innovation.

Consider the case in which the congruent employee is the one who values assignment relatively more than bonus pay. This employee is willing to give up more monetary compensation for a higher chance of working on a high-impact project. Suppose the firm's trade-off goes in the other direction, that is, it values motivating the employee to exert more effort on innovation more than it values assigning the congruent employee to the high-impact project. This is the case where marginally increasing effort on innovation brings high value to the firm. Below, we discuss the job design implications of our model in this case.

Under the above conditions, Proposition 7 shows that a contract that induces self-selection assigns the congruent employee less often to the high-impact project, but it offers this employee a higher bonus for success. Job designs with these characteristics are sometimes observed in technology companies. They allow engineers to allocate a percentage

[23] For instance, the Facebook CEO described this process as follows: "[A] lot of the acquisitions that we make at Facebook are, you know, we look at great entrepreneurs out there who are building things. And often, the acquisitions aren't even to really buy their company or what they're doing. It's to get the really talented people who are out there trying to build something cool and say, you know, if you joined Facebook, you could work on this completely different problem."Citation from Coyle and Polsky (2013) of interview by Charlie Rose (Nov. 7, 2011).

of their time to working on innovative projects.[24] Other employees may work full time on innovative projects within R&D units. The bonus for the engineers who develop successful projects is generally higher, as they may be promoted to lead their own team for that new product, or they may gain ownership over the new product. This type of promotion is generally a higher bonus compared to a full-time research scientist's compensation. Such job designs may not have been created with the stated purpose of inducing self-selection. Nevertheless, our results suggest that they have this additional consequence.

Another job design implication of our model under the conditions described in this example is that offering a fixed salary is sometimes necessary in order to achieve self-selection. We discuss two implementations of this contract feature below. First, our notion of fixed salary can be readily mapped to offering base pay, non-contingent on the output produced. This practice is widely used in technology companies and elsewhere. While it may also be implemented for motivations outside the scope of the model, its use may nevertheless have the side benefit of inducing the employee self-selection that benefits the firm.

Second, for the special case when the low-congruence employee is very unproductive on the high-impact project, another common implementation of a fixed salary is the use of severance pay. Contracts that include severance, golden parachutes, or 'pay-to-quit' provisions specify a payment made to the employee if he or she leaves the organization. In this case, the low-impact project can be interpreted as the firm's value from being able to replace the current employee. For instance, organizations usually have only one department chief for each department. Yet, in the case of an acquisition, the acquiring firm may have gained several qualified candidates to potentially head one of its departments. In that case, monetary compensation for quitting can be used as a tool for a non-congruent department chief to self-select out of the organization. This selection out of the firm is valuable, as the firm can then search for and appoint a congruent employee for that opened position. Our results

[24]This practice started (but has since been discontinued) at Google, with a rule to allow engineers to spend 20% of their time on innovative projects, and was afterwards adopted by other technology companies including LinkedIn, Apple, and Microsoft. A journalistic description of these programs can be found https://www.fastcompany.com/3015963/google-took-its-20-back-but-other-companies-are-making-employee-side-projects-work-for-them, accessed March 26, 2020.

therefore suggest that severance or 'pay-to-quit' incentives are beneficial even when taken up *before* an employee starts to work on a project. This complements existing insights in the literature, which have focused on using these contract features to influence the employee's choices during or after his or her work on projects (Dur and Schmittdiel, 2013; Almazan and Suarez, 2003; Manso, 2011; Inderst and Mueller, 2010).

6 Robustness and Extensions

Our model and results are not limited to the case of two project outcomes or two employee types. In the Online Appendix, we show that the analysis and intuition carry over to settings with multiple employee types or multiple project outcomes. We briefly discuss the insights from these extensions below.

Multiple Employee Types

In the Online Appendix C, we analyze the model with any number of employee types. We show that our results from Propositions 5 and 6 carry through. The very unproductive employee types are given a fixed salary and are assigned to the low-impact project. The sufficiently productive employee types are assigned to either project. As in the two-type case, when the firm's and the higher-type employee's preferences are in accord, both value assignment relatively more than incentives for effort. The optimal contract then reflects this preference: the probability of assignment increases in employee type, while the bonus for success decreases in employee type.

When the firm's and the higher-type employee's preferences are in dis-accord, the firm would like to offer a larger bonus for a higher type, while a higher type employee prefers contracts with more frequent assignment to the high-impact project. We show that the equivalent contract to that of Proposition 7 is, in fact, a special case of a more general type of contract. This contract involves a fixed salary only for employee types below a threshold type. For the employee types below the threshold, the results of Proposition 7 carry through. As the employee type increases, the contract offers a lower probability of assignment, a higher bonus, and a lower fixed payment. That is, the contract reflects the firm's preference. However, for the employee types above the threshold, as the employee type increases, the contract offers a higher probability of assignment and a lower bonus. That is, the contract reflects the employee's preference. The intuition is that for every increase in the bonus for a higher type, the firm must increase the fixed salary to the lowest type, since it is the lowest type who values the bonus the most relative to assignment. This is acceptable for the firm if the distance in terms of effort cost between the highest and the lowest types is not too large. If the discrepancy between the firm's preference and the employee's preference widens as we add more extreme types into the pool of employees, it becomes optimal to distort the contract at the top to save the agency cost at the bottom.

Multiple Project Outcomes

In the Online Appendix D, we analyze the case where the firm's high-impact project can yield more than two outcomes. If the firm could observe employee type, it would offer compensation only through the bonus, and only after the outcome with the highest hazard rate. When the firm cannot observe types, it must also induce self-selection. If both the firm's and the employee's preferences are in accord, then the firm can trade off the bonus for the best outcome with the probability of assignment, as in Proposition 6. If the firm and the employee's preferences are in dis-accord, then the firm may offer the L-type either a fixed salary or bonuses for intermediate outcomes. Note that the bonuses for intermediate outcomes play the same role as the fixed salary. They are not the most efficient ways to pay a rent to the L-type, but they are useful for inducing self-selection, as the L-type prefers the bonus from such outcomes relatively more than the H-type.

7 Concluding Remarks

We presented a model that speaks to the problem of re-assigning human capital after a major transition: the employees who started their careers under the old regime have private information about their congruence with the new regime. In many cases, whether in public bureaucracies or private firms under strict labor laws, firing the old employees and starting anew is not a viable option. This leaves management with the challenge of inducing employee self-selection and incentivizing effort post-selection.

Our findings show that the optimal contract requires offering employees a menu in which each contract trades off assignment to high-impact work against monetary compensation. One type of contract assigns the employee fully to one project but offers lower monetary compensation. The other contract specifies a smaller probability of assignment, equivalent to a fraction of time to be spent on a high-impact project, but with high rewards for the high-impact project's success. Understanding which employee gets which type of contract reduces to understanding how the firm and the employee each value selection versus incentives. If there is accord in their relative valuation of the two factors, then the contract will favor solving the issue that both the firm and the congruent employee value more. If this issue is selection, this means the congruent employee will be assigned to the high-impact project more often. If the issue is incentives, the congruent employee is the one offered a higher bonus for success. If there is dis-accord in the relative valuation of selection versus incentives by the firm versus the employee, the resolution comes in the form of a contract that reflects the firm's preferences, but that adds a fixed salary for the low-congruence employee.

Our model and results extend the literature on contracting under adverse selection and moral hazard in one key dimension: we give the organization access to two valuable projects, only one of which requires unobservable effort. This characteristic, natural for many organizations, leads to the result that the optimal contract can, in fact, induce employees to self-select. The resulting job design implications enhance our understanding of the effects of some relatively common, albeit at first puzzling, contract features encountered in practice.

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A Preliminaries

Although θ is binary, we assume that H and L are real numbers and all the functions are well defined for $\theta \in [L, H]$, so that we can take derivatives with respect to θ. In the proofs, the following concavity/convexity results will be useful:

$$\begin{aligned}
e_\theta(\theta, b) &= -\frac{c_{\theta e}(\theta, e(\theta, b))}{c_{ee}(\theta, e(\theta, b))}; \\
e_{\theta b}(\theta, b) &= -q_1 \frac{c_{ee\theta}(\theta, e(\theta, b))}{[c_{ee}(\theta, e(\theta, b))]^2} + \frac{c_{eee}(\theta, e(\theta, b))\, c_{\theta e}(\theta, e(\theta, b))}{[c_{ee}(\theta, e(\theta, b))]^3} \geq 0 \text{ by (1)}; \\
e_{bb}(\theta, b) &= \frac{-q_1 e(\theta, b)\, c_{eee}(\theta, e(\theta, b))}{[c_{ee}(\theta, e(\theta, b))]^2} \leq 0 \text{ by (1)}; \\
S_\theta(\theta, b) &= (q_1 - c_e(\theta, e(\theta, b)))e_\theta(\theta, b) - c_\theta(\theta, e(\theta, b)); \\
S_b(\theta, b) &= q_1(1-b)\, e_b(\theta, b) \geq 0; \\
S_{b\theta}(\theta, b) &= q_1(1-b)\, e_{\theta b}(\theta, b) \geq 0; \\
S_{bb}(\theta, b) &= -q_1(e(\theta, b))\, e_b(\theta, b) + q_1 e(\theta, b)(1-b)\, e_{bb}(\theta, b) \leq 0; \\
V_\theta(\theta, b) &= -c_\theta(\theta, b) > 0; \\
V_b(\theta, b) &= q_0 + q_1 e(\theta, b); \\
V_{b\theta}(\theta, b) &= q_1 e_\theta(\theta, b) = -q_1 \frac{c_{\theta e}(\theta, e(\theta, b))}{c_{ee}(\theta, e(\theta, b))} \geq 0; \\
V_{bb}(\theta, b) &= q_1 e_b(\theta, b) \geq 0; \\
\pi_{bb}(\theta, b) &= S_{bb}(\theta, b) - V_{bb}(\theta, b) \leq 0. \qquad (15)
\end{aligned}$$

B Proofs

B.1 Proof of Proposition 1

The firm's problem is

$$\max_{\substack{p_H, p_L, e_H, e_L, \\ b_H(1), b_H(0), r_H, \\ b_L(1), b_L(0), r_L}} \begin{array}{l} \mu_H \cdot (p_H \cdot (q(e_H) \cdot (1 - b_H(1)) - (1 - q(e_H)) \cdot b_H(0)) + (1 - p_H) \cdot W - r_H) \\ + (1 - \mu_H) \cdot (p_L (q(e_L) \cdot (1 - b_L(1)) - (1 - q(e_L)) \cdot b_L(0)) + (1 - p_L) \cdot W - r_L) \end{array}$$

subject to

$$q(e_H) \cdot b_H(1) + (1 - q(e_H)) \cdot b_H(0) - c(e_H, H) \geq 0,$$

$$q(e_L) \cdot b_L(1) + (1 - q(e_L)) \cdot b_L(0) - c(e_L, L) \geq 0,$$

$$p_H \cdot (q(e_H) \cdot b_H(1) + (1 - q(e_H)) \cdot b_H(0) - c(e_H, H)) + r_H \geq r_H,$$

$$p_L \cdot (q(e_L) \cdot b_L(1) + (1 - q(e_L)) \cdot b_L(0) - c(e_L, L)) + r_L \geq r_L.$$

Clearly, $r_H = r_L = 0$ is optimal. In addition, by the limited liability, the last two constraints are always satisfied. Moreover, by denoting $b_\theta = q(e_\theta) \cdot b_\theta(1) + (1 - q(e_\theta)) \cdot b_\theta(0)$, the firm's problem becomes

$$\max_{\substack{p_H, p_L, e_H, e_L, \\ b_H, b_L}} \begin{array}{c} \mu_H \cdot (p_H \cdot (q(e_H) - b_H) + (1 - p_H) \cdot W) \\ + (1 - \mu_H) \cdot (p_L (q(e_L) - b_L) + (1 - p_L) \cdot W - r_L) \end{array}$$

subject to

$$b_H - c(e_H, H) \geq 0,$$

$$b_L - c(e_L, L) \geq 0.$$

Clearly, the two constraints are binding. Hence, the firm's problem becomes

$$\max_{p_H, p_L} \begin{array}{c} \mu_H \cdot (p_H \cdot \max_{e_H} (q(e_H) - c(e_H, H)) + (1 - p_H) \cdot W) \\ + (1 - \mu_H) \cdot (p_L \cdot \max_{e_L} (q(e_L) - c(e_L, L)) + (1 - p_L) \cdot W - r_L) . \end{array}$$

Therefore, the implemented effort level for type θ is e_θ^{FB} and $S(\theta, 1) \equiv \max_{e_\theta} (q(e_\theta) - c(e_\theta, \theta))$ by definition. Since Assumption 1 implies that $S(H, 1) > W$, the result holds.

B.2 Proof of Proposition 2

$p_H = 1$ and $p_L = 0$ follows immediately from Assumption 1. Substituting $p_H = 1$ and $p_L = 0$ in (6) and solving for the firm's problem without the incentive compatibility constraints immediately yields $b_H > 0$ and $b_L = r_L = r_H = 0$.

B.3 Proof of Proposition 3

The firm's problem is

$$\max_{\substack{p_H, p_L, e_H, e_L, \\ b_H(1), b_H(0), r_H, \\ b_L(1), b_L(0), r_L}} \begin{array}{c} \mu_H \cdot (p_H \cdot (q(e_H) \cdot (1 - b_H(1)) - (1 - q(e_H)) \cdot b_H(0)) + (1 - p_H) \cdot W - r_H) \\ + (1 - \mu_H) \cdot (p_L \cdot (q(e_L) \cdot (1 - b_L(1)) - (1 - q(e_L)) \cdot b_L(0)) + (1 - p_L) \cdot W - r_L) \end{array}$$

subject to

$$q(e_H) \cdot b_H(1) + (1 - q(e_H)) \cdot b_H(0) - c(e_H, H) \geq 0,$$

$$q(e_L) \cdot b_L(1) + (1 - q(e_L)) \cdot b_L(0) - c(e_L, L) \geq 0,$$

$$\begin{aligned} & p_H \cdot (q(e_H) \cdot b_H(1) + (1 - q(e_H)) \cdot b_H(0) - c(e_H, H)) + r_H \\ & \qquad \geq \max\{p_L \cdot (q(e_L) \cdot b_L(1) + (1 - q(e_L)) \cdot b_L(0) - c(e_L, H)) + r_L, r_L\}, \end{aligned} \quad (16)$$

$$p_L \cdot (q(e_L) \cdot b_L(1) + (1 - q(e_L)) \cdot b_L(0) - c(e_L, L)) + r_L$$
$$\geq \max \{p_H \cdot (q(e_H) \cdot b_H(1) + (1 - q(e_H)) \cdot b_H(0) - c(e_H, L)) + r_H, r_H\}. \quad (17)$$

Writing $q(e_H) \cdot b_H(1) + (1 - q(e_H)) \cdot b_H(0) = b_H$ and $q(e_L) \cdot b_L(1) + (1 - q(e_L)) \cdot b_L(0) - c(e_L, L) = b_L$, the firm's problem becomes

$$\max_{\substack{p_H, p_L, e_H, e_L, \\ b_H, r_H, b_L, r_L}} \begin{array}{c} \mu_H \cdot (p_H \cdot (q(e_H) - b_H - W) - r_H) \\ + (1 - \mu_H) \cdot (p_L \cdot (q(e_L) - b_L - W) - r_L) \end{array}$$

subject to

$$b_H - c(e_H, H) \geq 0,$$
$$b_L - c(e_L, L) \geq 0,$$
$$p_H \cdot (b_H - c(e_H, H)) + r_H \geq \max \{p_L \cdot (b_L - c(e_L, H)) + r_L, r_L\}, \quad (18)$$
$$p_L \cdot (b_L - c(e_L, L)) + r_L \geq \max \{p_H \cdot (b_H - c(e_H, L)) + r_H, r_H\}. \quad (19)$$

Since $b_L - c(e_L, H) \geq b_L - c(e_L, L) \geq 0$, we can replace (18) with

$$p_H \cdot (b_H - c(e_H, H)) + r_H \geq p_L \cdot (b_L - c(e_L, H)) + r_L.$$

We now relax the problem by ignoring (19):

$$\max_{\substack{p_H, p_L, e_H, e_L, \\ b_H, r_H, b_L, r_L}} \begin{array}{c} \mu_H \cdot (p_H \cdot (q(e_H) - b_H - W) - r_H) \\ + (1 - \mu_H) \cdot (p_L \cdot (q(e_L) - b_L - W) - r_L) \end{array}$$

subject to

$$b_H - c(e_H, H) \geq 0,$$
$$b_L - c(e_L, L) \geq 0,$$
$$p_H \cdot (b_H - c(e_H, H)) + r_H \geq p_L \cdot (b_L - c(e_L, H)) + r_L.$$

In this problem, we have $b_L = c(e_L, L)$ since lower b_L improves the objective and relaxes all the constraints but $b_L - c(e_L, L) \geq 0$. Moreover, $r_L = 0$ since lower r_L improves the objective and relaxes all the constraints. Hence, the problem becomes

$$\max_{\substack{p_H, p_L, e_H, e_L, \\ b_H, r_H, r_L}} \begin{array}{c} \mu_H \cdot (p_H \cdot (q(e_H) - b_H - W) - r_H) \\ + (1 - \mu_H) \cdot p_L \cdot (q(e_L) - c(e_L, L) - W) \end{array}$$

subject to

$$b_H - c(e_H, H) \geq 0$$

and

$$p_H \cdot (b_H - c(e_H, H)) + r_H \geq p_L \cdot (c(e_L, L) - c(e_L, H)). \quad (20)$$

If $p_L = 0$, then (20) is not binding. Hence, $r_H = r_L = 0$ and $b_H = c(e_H, H)$. The firm's payoff is

$$\max_{p_H\in[0,1],e_H\in[0,1]} \mu_H \cdot [p_H \cdot (q(e_H) - c(e_H, H)) + (1-p_H)\cdot W] + (1-\mu_H)\cdot W. \tag{21}$$

If $p_L > 0$, then (20) is binding and

$$p_H \cdot b_H + r_H = p_H \cdot c(e_H, H) + p_L \cdot (c(e_L, L) - c(e_L, H)).$$

Hence, the firm's payoff is

$$\max_{p_H,p_L,e_H,e_L} \begin{array}{c} \mu_H \cdot \left[\ p_H \cdot (q(e_H) - c(e_H, H)) + (1-p_H)\cdot W - p_L \cdot (c(e_L, L) - c(e_L, H))\ \right] \\ +(1-\mu_H)\cdot [p_L \cdot (q(e_L) - c(e_L, L)) + (1-p_L)\cdot W] \end{array}$$

Note that we can separate this problem into the two steps:

$$\max_{p_H\in[0,1],e_H\in[0,1]} p_H \cdot (q(e_H) - c(e_H, H)) + (1-p_H)\cdot W \tag{22}$$

and

$$\max_{p_L\in[0,1],e_L\in[0,1]} (1-\mu_H)\cdot[p_L \cdot (q(e_L) - c(e_L, L)) + (1-p_L)\cdot W] - \mu_H \cdot p_L \cdot (c(e_L, L) - c(e_L, H)). \tag{23}$$

By Assumption 1, even when the effort is not observable, it is optimal to allocate the high-type employee to the high-impact project. Hence, in both (21) and (22), we have $p_H = 1$. In total, the optimal contract is characterized as follows:

$$e_H = \arg\max_{e_H\in[0,1]} q(e_H) - c(e_H, H).$$

1. If (22) yields $p_L > 0$, then it is optimal to have $p_L > 0$. We have $b_H = c(e_H, H) + p_L (c(e_L, L) - c(e_L, H))$ and $r_H = r_L = 0$.

   In this case, we set $r_H = 0$. For the relaxed problem, any pair (b_H, r_H) such that

   $$b_H \geq c(e_H, H) \text{ and } b_H + r_H = c(e_H, H) + p_L \cdot (c(e_L, L) - c(e_L, H))$$

   satisfies the condition. However, when we consider the original problem, reducing r_H and increasing b_H by $\frac{1}{p_H}$ relaxes the incentive compatibility constraint of the low-type, since it becomes more costly to obtain the rent from pretending to be the high-type.

2. Otherwise, $p_L = 0$, $r_H = r_L = 0$, and $b_H = c(e_H, H)$.

We now verify that the omitted constraint (19) is satisfied: If $p_L = 0$, then

$$\begin{aligned} & \max\{p_H \cdot (b_H - c(e_H, L)) + r_H, r_H\} \\ = \ & b_H - c(e_H, L) \\ = \ & c(e_H, H) - c(e_H, L) < 0. \end{aligned}$$

If $p_L > 0$, we have

$$\begin{aligned}
& \max\left\{p_H \cdot \left(b_H - c\left(e_H, L\right)\right) + r_H, r_H\right\} \\
= \ & b_H - c\left(e_H, L\right) \\
= \ & c\left(e_H, H\right) + p_L\left(c\left(e_L, L\right) - c\left(e_L, H\right)\right) - c\left(e_H, L\right) \\
\leq \ & c\left(e_H, H\right) - c\left(e_H, L\right) + c\left(e_L, L\right) - c\left(e_L, H\right).
\end{aligned}$$

Since $c_{e\theta} \leq 0$, it suffices to prove that $e_H > e_L$. Note that, for the maximization problem (22), given the optimal p_L, e_L satisfies

$$e_L \in \arg\max_{e_L \in [0,1]} \left(1 - \mu_H\right) \cdot p_L \cdot \left(q\left(e_L\right) - c\left(e_L, L\right)\right) - \mu_H \cdot p_L \cdot \left(c\left(e_L, L\right) - c\left(e_L, H\right)\right).$$

Since $c_{e\theta} \leq 0$, we have $e_L < \arg\max q\left(e_L\right) - c\left(e_L, L\right)$. Again since $c_{e\theta} \leq 0$, we have

$$e_L < \arg\max q\left(e_L\right) - c\left(e_L, L\right) < \arg\max q\left(e_H\right) - c\left(e_H, H\right) = e_H,$$

as desired.

B.4 Proof of Lemma 1

B.4.1 Proof that (IC_L) holds with equality

If $r_L = 0$ and (IC_L) does not hold with equality, then the firm would simply reduce p_L, given Assumption 1. If $r_L > 0$ and (IC_L) is not binding, then the firm would simply reduce r_L (regardless of Assumption 1).

B.4.2 Proof that (IC_H) holds with equality if $p_L > 0$ and $r_L > 0$

Suppose $p_L > 0, r_L > 0$ and (IC_H) is not binding. Then, reducing $r_L > 0$ and increasing b_L to keep $p_L \cdot V\left(L, b_L\right) + r_L$ fixed improves the firm's payoff and satisfies the incentive compatibility constraint, since the social welfare is improved and the rent paid to the L-type is fixed.

B.4.3 Proof that $r_H = 0$

Assume $r_H > 0$. Note that (IC_H) is binding since otherwise the firm would reduce r_H:

$$p_H \cdot V\left(H, b_H\right) + r_H = p_L \cdot V\left(H, b_L\right) + r_L. \tag{24}$$

From (6), the firm's payoff from the H-type is

$$\begin{aligned}
& p_H \cdot \left(S\left(H, b_H\right) - V\left(H, b_H\right)\right) + \left(1 - p_H\right) \cdot W - r_H \\
& \qquad = p_H \cdot \left(S\left(H, b_H\right) - W\right) + W - p_L \cdot V\left(H, b_L\right) - r_L.
\end{aligned} \tag{25}$$

Take a pair $(\Delta_r, \Delta_b) \in \mathbb{R}_+^2$ such that offering $r_H - \Delta_r$ and $b_H + \Delta_b$ keeps the equality (24):

$$p_H \cdot V(H, b_H + \Delta_b) + r_H - \Delta_r = p_L \cdot V(H, b_L) + r_L. \tag{26}$$

Offering $r_H - \Delta_r$ and $b_H + \Delta_b$ improves the firm's payoff since the social welfare is improved by (15) and the rent paid for the H-type is fixed by (26). Moreover, (IC_L) is satisfied:

$$\begin{aligned} p_H \cdot V(L, b_H + \Delta_b) + r_H - \Delta_r &= p_H \cdot V(L, b_H) + r_H + p_H \cdot V(L, b_H + \Delta_b) - p_H \cdot V(L, b_H) - \Delta_r \\ &\le p_H \cdot V(L, b_L) + r_L + p_H \cdot V(L, b_H + \Delta_b) - p_H \cdot V(L, b_H) - \Delta_r \\ &\le p_H \cdot V(L, b_L) + r_L + p_H \cdot V(H, b_H + \Delta_b) - p_H \cdot V(H, b_H) - \Delta_r. \end{aligned} \tag{27}$$

The last line follows since $V_{\theta b}(\theta, b) \ge 0$ by (15):

$$\begin{aligned} & p_H \cdot V(H, b_H + \Delta_b) - p_H \cdot V(H, b_H) \\ &\quad = p_H \cdot V(L, b_H + \Delta_b) - p_H \cdot V(L, b_H) + p_H \cdot \int_{\theta=L}^{H} \left(V_\theta(\theta, b_H + \Delta_b) - V_\theta(\theta, b_H)\right) d\theta \\ &\quad = p_H \cdot V(L, b_H + \Delta_b) - p_H \cdot V(L, b_H) + p_H \cdot \int_{\theta=L}^{H} \int_{b=b_H}^{b_H + \Delta_b} V_{\theta b}(\theta, b)\, db d\theta \\ &\quad \ge p_H \cdot V(L, b_H + \Delta_b) - p_H \cdot V(L, b_H). \end{aligned} \tag{28}$$

By (26), this inequality implies

$$p_H \cdot V(L, b_H + \Delta_b) + r_H - \Delta_r \le p_L \cdot V(L, b_L) + r_L.$$

Therefore, we have $r_H = 0$.

B.4.4 Proof that either $p_L = 1$ or $p_H = 1$

Assume next $p_H < 1$ and $p_L < 1$. From (6), the firm's objective is

$$\begin{aligned} & \mu_H \cdot [p_H \cdot \pi(H, b_H) + (1 - p_H) \cdot W - r_H] + \mu_L \cdot [p_L \cdot \pi(L, b_L) + (1 - p_L) \cdot W - r_L] \\ &\quad = \mu_H \cdot (p_H \cdot [\pi \cdot (H, b_H) - W] - r_H) + \mu_L \cdot [p_L \cdot (\pi \cdot (L, b_L) - W) - r_L] + W. \end{aligned} \tag{29}$$

We have

$$\mu_H \cdot [p_H(\pi \cdot (H, b_H) - W) - r_H] + \mu_L \cdot [p_L(\pi \cdot (L, b_L) - W) - r_L] > 0, \tag{30}$$

since otherwise $p_H = p_L = r_H = r_L = 0$ would be optimal (recall that we consider the case when $\max\{p_H, p_L\} > 0$).

Offering $(k \cdot p_H, b_H, k \cdot r_H)$ and $(k \cdot p_L, b_L, k \cdot r_L)$ with $k > 1$ increases the firm's profit:

$$\begin{aligned} &\mu_H \cdot [k \cdot p_H \cdot \pi(H, b_H) + (1 - k \cdot p_H)W - k \cdot r_H] + \mu_L \cdot [k \cdot p_L \pi(L, b_L) + (1 - k \cdot p_L) \cdot W - r_L] \\ &\quad = k \cdot [\mu_H \cdot [p_H [\pi \cdot (H, b_H) - W] - r_H] + \mu_L \cdot (p_L \cdot (\pi(L, b_L) - W) - r_L)] + W. \end{aligned} \tag{31}$$

Since (IC_H) and (IC_L) are satisfied, this is a profitable deviation, which is a contradiction.

B.5 Proof of Proposition 4

If $r_L = 0$, then (IC_H) implies $p_H \geq p_L$ if $b_H \leq b_L$, and (IC_L) implies $p_L \geq p_H$ if $b_H \geq b_L$. If $r_L > 0$, then both (IC_H) and (IC_L) bind from Lemma 1. Hence

$$p_H \cdot V(H, b_H) = p_L \cdot V(H, b_L) + r_L; \tag{32}$$

$$p_H \cdot V(L, b_H) = p_L \cdot V(L, b_L) + r_L, \tag{33}$$

and so

$$\frac{p_H}{p_L} = \frac{V(H, b_L) - V(L, b_L)}{V(H, b_H) - V(L, b_H)}. \tag{34}$$

Hence, $p_H \geq p_L$ if and only if

$$V(H, b_L) - V(H, b_H) - (V(L, b_L) - V(L, b_H)) \geq 0. \tag{35}$$

Since

$$V(H, b_L) - V(H, b_H) - (V(L, b_L) - V(L, b_H)) = \int_{\theta=L}^{H} \int_{b=b_H}^{b_L} V_{\theta b}(\theta, b)\, db d\theta, \tag{36}$$

and $V_{\theta b}(\theta, b) = q_1 e_\theta(\theta, b) > 0$ by (15), we have $p_H \geq p_L$ if $b_H \leq b_L$, and $p_L \geq p_H$ if $b_H \geq b_L$.

B.6 Proof of Lemma 2

The firm's problem with $W = 0$ is

$$\max_{p_H, p_L, b_H \geq 0, b_L \geq 0, r_L \geq 0} \mu_H \cdot p_H \cdot \pi(H, b_H) + \mu_L \cdot [p_L \cdot \pi(L, b_L) - r_L] \tag{37}$$

subject to

$$p_H \cdot V(H, b_H) \geq p_L \cdot V(H, b_L) + r_L, \tag{IC_H}$$

$$p_L \cdot V(L, b_L) + r_L \geq p_H \cdot V(L, b_H), \tag{IC_L}$$

where the constraints incorporate the result of Lemma 1 that $r_H = 0$ for any value of W.

To prove the result, by Proposition 4, it suffices to show that $p_H = p_L = 1$.

Case 1. Suppose both (IC_H) and (IC_L) are not binding. Then, the optimal contract would be $b_\theta = \arg\max_b q\left(e\left(\theta, b\right)\right) \cdot (1-b)$ and $p_H = p_L = 1$.

Case 2. If (IC_H) is not binding but (IC_L) is binding, then $p_L = 1$ and $r_L = 0$ since Lemma 1 holds for any value of W. Hence, (IC_L) implies

$$V\left(L, b_L\right) = p_H \cdot V\left(L, b_H\right) = p_H \cdot V\left(L, b_H\right) + (1-p_H) \cdot V\left(L, 0\right) \geq V\left(L, p_H b_H\right),$$

since V is convex in b by (15). Suppose instead the firm offers $\mathcal{C}_H = (1, p_H b_H, 0)$ to the H-type. IC_L holds by (IC_L). At the same time, since π is concave in b by (15), we have

$$\pi\left(H, p_H b_H\right) \geq p_H \cdot \pi\left(H, b_H\right) + (1-p_H) \cdot \pi\left(H, 0\right) \geq p_H \cdot \pi\left(H, b_H\right). \tag{38}$$

This contract is profit improving. Hence, $p_H = p_L = 1$. Note that the last inequality in (38) uses the fact that $\pi\left(H, 0\right) \geq 0$. With $W > 0$, we would have $\pi\left(H, 0\right) - W$ instead, which could be negative, and the conclusion might not hold.

Case 3. If (IC_L) is not binding but (IC_H) is binding, then $p_H = 1$ and $r_L = 0$. Hence, IC_H is written as $p_L \cdot V\left(H, b_L\right) = V\left(H, b_H\right)$. Suppose instead the firm offers $\mathcal{C}_L = (1, p_L b_L, 0)$ to the L-type. The same proof as above implies that this contract is incentive compatible and profit improving. Hence, $p_H = p_L = 1$.

Case 4. Suppose both (IC_H) and (IC_L) are binding. Since Proposition 4 holds for any value of W, we have $[1 = p_H \geq p_L$ and $b_H \leq b_L]$ or $[p_H \leq p_L = 1$ and $b_H \geq b_L]$. Suppose $1 = p_H \geq p_L$ and $b_H \leq b_L$. Since $V\left(L, b_H\right) = p_L \cdot V\left(L, b_L\right) + r_L \geq V\left(L, p_L b_L\right) + r_L$ implies $b_H \geq p_L b_L$, we have

$$\begin{aligned}
\pi\left(L, b_H\right) &= S\left(L, b_H\right) - V\left(L, b_H\right) \\
&= S\left(L, b_H\right) - p_L \cdot V\left(L, b_L\right) - r_L, \text{ by } IC_L \\
&\geq S\left(L, p_L b_L\right) - p_L \cdot V\left(L, b_L\right) - r_L, \text{by } b_H \geq p_L \cdot b_L \\
&\geq p_L \cdot S\left(L, b_L\right) + \underbrace{(1-pL) \cdot S\left(L, 0\right)}_{\geq 0} - p_L \cdot V\left(L, b_L\right) - r_L, \text{ by concavity of } S \text{ in } b \\
&\geq p_L \cdot \pi\left(L, b_L\right) - r_L.
\end{aligned}$$

Hence, it is profit-improving to offer $(p, b, r) = (p_H, b_H, 0)$ to both players.

Suppose next $p_H \leq p_L = 1$ and $b_H \geq b_L$. By the symmetric proof, if $r_L = 0$, then we have $b_L \geq p_H \cdot b_H$;[25] and if $b_L \geq p_H \cdot b_H$, then it is profitable to offer $(p, b, r) = (p_L, b_L, 0)$ to both players. Hence, it remains to consider the case in which $r_L > 0$ and $b_L \leq p_H \cdot b_H$. Suppose the firm offers $(p, b, r) = (1, p_H b_H, 0)$ to both players. Since S is concave in b and

[25] The difference from the previous case is that, since $r_L > 0$, $V\left(H, b_L\right) + r_L = p_H \cdot V\left(H, b_H\right) \geq V\left(L, p_H b_H\right)$ does not imply $b_L \geq p_H \cdot b_H$.

V is convex in b, the profit from the H-type increases:

$$\begin{aligned}
&S(H, p_H b_H) - V(H, p_H b_H) \\
&\qquad \geq p_H \cdot S(H, b_H) + (1 - p_H) \cdot \underbrace{S(H, 0)}_{\geq 0} - p_H \cdot V(H, b_H) - (1 - p_H) \cdot \underbrace{V(H, 0)}_{=0} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad \geq p_H \cdot S(H, b_H) - p_H \cdot V(H, b_H). \quad (39)
\end{aligned}$$

The profit from the L-type also increases since

$$\begin{aligned}
\pi(L, p_H b_H) &= S(L, p_H b_H) - V(L, p_H b_H) \\
&\geq S(L, b_L) - V(L, p_H b_H), \text{ since } b_L \leq p_H b_H \\
&\geq S(L, b_L) - p_H \cdot V(L, b_H) - (1 - p_H) \cdot \underbrace{V(L, 0)}_{=0}, \text{ since } V \text{ is convex in } b \\
&= S(L, b_L) - V(L, b_L) - r_L, \text{ by } IC_L \\
&= \pi(L, b_L) - r_L.
\end{aligned}$$

Hence, it is profit-improving to offer $(p, b, r) = (1, p_H b_H, 0)$ to both players.

B.7 Proof of Proposition 5

Given Lemma 1, it suffices to prove that $p_L = 0$ is optimal if and only if $S(L, 1) - W \leq 0$.

The proof of $p_L = 0$ implying $S(L, 1) - W \leq 0$: Suppose $p_L = 0$ in the optimal contract. Then, we have

$$r_L = \underbrace{p_H \cdot V(L, b_H)}_{\text{since } IC_L \text{ binds}} < p_H \cdot V(H, b_H). \quad (40)$$

Hence, (IC_H) is not binding.

Since (IC_L) binds, it follows that $r_L = p_H \cdot V(L, b_H) - p_L \cdot V(L, b_L)$. Substituting r_L into the firm's problem,

$$\max_{p_H, p_L, b_H, b_L} \mu_H \cdot p_H \cdot [q(e(H, b_H)) \cdot (1 - b_H) - W] + \mu_L \cdot [p_L \cdot q(e(L, b_L)) \cdot (1 - b_L) - p_L \cdot W - r_L] \quad (41)$$

$\Leftrightarrow$

$$\begin{aligned}
&\max_{p_H, b_H} \mu_H \cdot p_H \cdot [q(e(H, b_H)) \cdot (1 - b_H) - W] - p_H \cdot \mu_L V(L, b_H) \\
&\qquad\qquad\qquad + \mu_L \cdot \max_{p_L} p_L \cdot \max_{b_L} [q(e(L, b_L)) - c(L, e(L, b_L)) - W]. \quad (42)
\end{aligned}$$

Only if $\max_{b_L} [q(e(L, b_L)) - c(L, e(L, b_L)) - W] \leq 0$, the optimal p_L is equal to 0.

The proof of $S(L, 1) - W \leq 0$ implying $p_L = 0$: Guess that (IC_H) is slack. As above, the firm's problem is equivalent to (42). If $S(L,1)-W \leq 0$, then the optimal p_L is equal to 0. Again, with $p_L = 0$, (IC_L) implies (IC_H) as (40). Hence, the guess of (IC_H) being slack is verified.

B.8 Complementary Lemmas

Lemma 3 *The employee's indifference curves exhibit increasing preference for assignment (bonus) for all (p,b) if and only if $V(\theta,b)$ is log-submodular (log-supermodular) in (θ,b). The firm's indifference curves exhibit increasing preference for assignment (bonus) if and only if $S(\theta,b)-W$ is log-submodular (log-supermodular) in (θ,b).*

The following lemma pins down the shape of the optimal contract given log-submodularity (or log-supermodularity) of V and a binding constraint (IC_L):

Lemma 4 *If (IC_L) holds with equality, then,*

1. *If V is log-submodular, then $b_H > b_L$ and $p_H < p_L$ if $r_L > 0$; and $b_H \leq b_L$ and $p_H \geq p_L$ if $r_L = 0$.*
2. *If V is log-supermodular, then $b_H < b_L$ and $p_H > p_L$ if $r_L > 0$; and $b_H \geq b_L$ and $p_H \leq p_L$ if $r_L = 0$.*

B.8.1 Proofs of Complementary Lemmas

Proof of Lemma 3 By definition, a function $F(\theta,b)$ is log-supermodular (log-submodular) if the following property holds:

$$F(\theta,b)\cdot F(\theta',b') - F(\theta',b)\cdot F(\theta,b') \geq (\leq) 0$$

$$\text{if and only if} \tag{43}$$

$$(\theta-\theta')\cdot(b-b') \geq (\leq) 0. \tag{44}$$

If $\log F(\theta,b)$ is twice-differentiable, then it is log-supermodular (log-submodular) whenever

$$\frac{d^2}{d\theta db}\log F(\theta,b) \geq (\leq) 0. \tag{45}$$

.

For any arbitrary contract $\mathcal{C}_H$, consider the set of contracts $\mathcal{C}$ with $r=0$ that would make type θ indifferent between $\mathcal{C}_H$ and $\mathcal{C}$. On the indifference curve obtained in this way, the marginal rate of substitution of b for p satisfies

$$V(\theta,b)\,dp + pV_b(\theta,b)\,db = 0 \Leftrightarrow -\frac{dp}{db} = p\frac{V_b(\theta,b)}{V(\theta,b)}.$$

Then, $\left(-\frac{dp}{db}\right)\big|_{\theta=H} \leq (\geq) \left(-\frac{dp}{db}\right)\big|_{\theta=L}$ is equivalent to $p\frac{V_b(\theta,b)}{V(\theta,b)}\Big|_{\theta=H} \leq (\geq)\, p\frac{V_b(\theta,b)}{V(\theta,b)}\Big|_{\theta=L}$. With $V(\theta,b)$ differentiable with respect to θ and b, this implies

$$\frac{\partial}{\partial\theta}\frac{V_b(\theta,b)}{V(\theta,b)} \leq (\geq)\, 0. \tag{46}$$

Thus we have

$$\frac{\partial}{\partial\theta}\frac{V_b(\theta,b)}{V(\theta,b)} = \frac{V_{b\theta}(\theta,b)\cdot V(\theta,b) - V_b(\theta,b)\cdot V_\theta(\theta,b)}{(V(\theta,b))^2} = \frac{d^2}{d\theta db}\log V(\theta,b) \leq (\geq)\, 0. \tag{47}$$

The analysis for $S(\theta,b) - W$ is analogous.

Proof of Lemma 4 We present the proof focusing on log-submodular V, since the argument for log-supermodular V is symmetric.

Part 1: If $r_L > 0$, then $b_H > b_L$ and $p_H < p_L$. Since Lemma 1 implies both (IC_H) and (IC_H) are binding, rearranging them yields

$$\begin{cases} p_H = p_L \cdot \dfrac{V(H,b_L) - V(L,b_L)}{V(H,b_H) - V(L,b_H)}; \\ p_H \cdot V(L,b_H) = p_L \cdot V(L,b_L) + r_L. \end{cases} \tag{48}$$

Since $r_L > 0$, we have $p_H V(L,b_H) > p_L V(L,b_L)$, and so

$$\frac{V(H,b_L) - V(L,b_L)}{V(H,b_H) - V(L,b_H)} \cdot V(L,b_H) > V(L,b_L) \Leftrightarrow V(H,b_L)\cdot V(L,b_H) > V(H,b_H)\cdot V(L,b_L). \tag{49}$$

Hence we have $b_H > b_L$ if V is log-submodular. The system of equations (48) implies $p_H < p_L$.

Part 2: If $r_L = 0$, then $b_H \leq b_L$ and $p_L \geq p_H$. Since (IC_L) is satisfied with equality, it follows that $p_H \cdot V(L,b_H) = p_L \cdot V(L,b_L)$. If V is log-submodular and $b_H > b_L$, then $V(H,b_H)/V(L,b_H) < V(H,b_L)/V(L,b_L)$, and so $p_H \cdot V(H,b_H) < p_L \cdot V(H,b_L)$. This contradicts to ($IC_L$). Hence $b_H \leq b_L$, and so $p_H/p_L = V(L,b_L)/V(L,b_H) \geq 1$.

B.9 Proof of Propositions 6 and 7

If the firm offers $\mathcal{C}_H$ to the L-type, its profit changes by

$$\begin{aligned} 0 \geq{} & [p_H \cdot \pi(L,b_H) + (1-p_H)\cdot W] - [p_L \cdot \pi(L,b_L) - r_L + (1-p_L)\cdot W] \\ & \geq p_H \cdot [S(L,b_H) - W] - p_L \cdot [S(L,b_L) - W], \text{ by } (IC_L). \end{aligned} \tag{50}$$

On the other hand, if it offers $\mathcal{C}_L$ to the H-type, its profit changes by

$$\begin{aligned} 0 &\geq \left[p_L \cdot \pi\left(H, b_L\right) - r_L + \left(1-p_L\right) \cdot W\right] - \left[p_H \cdot \pi\left(H, b_H\right) + \left(1-p_H\right) \cdot W\right] \\ &\geq p_L \cdot \left[S\left(H, b_L\right) - W\right] - p_H \cdot \left[S\left(H, b_H\right) - W\right] \text{ by } \left(IC_H\right). \end{aligned} \tag{51}$$

Rearranging, we obtain

$$p_H \cdot \left[S\left(H, b_H\right) - S\left(L, b_H\right)\right] \geq p_L \cdot \left[S\left(H, b_L\right) - S\left(L, b_L\right)\right]. \tag{52}$$

Since (50) implies $p_H \leq p_L \cdot \left[S\left(L, b_L\right) - W\right] / \left[S\left(L, b_H\right) - W\right]$, we obtain

$$\begin{gathered} \frac{S\left(L, b_L\right) - W}{S\left(L, b_H\right) - W} \cdot \left[\left(S\left(H, b_H\right) - W\right) - \left(S\left(L, b_H\right) - W\right)\right] \geq \left(S\left(H, b_L\right) - W\right) - \left(S\left(L, b_L\right) - W\right) \\ \Leftrightarrow \\ \left(S\left(L, b_L\right) - W\right) \cdot \left(S\left(H, b_H\right) - W\right) \geq \left(S\left(L, b_H\right) - W\right) \cdot \left(S\left(H, b_L\right) - W\right). \end{gathered} \tag{53}$$

Hence if the net social welfare is log-supermodular (or log-submodular), then $b_H \geq b_L$ (or $b_H \leq b_L$). Given Lemma 4 and $V(\theta, b)$ log-submodular, it follows that $r_L > 0$ if and only if $b_H > b_L$.

B.10 Proof of Corollary 2

By Proposition 6, we have $r_L = 0$ and $b_H \leq b_L$. Lemmas 1 and 4 imply that $p_H = 1$. Since (IC_L) is satisfied with equality by Assumption 1, we have $V\left(L, b_H\right) = p_L V\left(L, b_L\right)$. Since V is log-submodular and $b_H \leq b_L$, we have $V\left(H, b_H\right) / V\left(L, b_H\right) \geq V\left(H, b_L\right) / V\left(L, b_L\right)$, and so $V\left(H, b_H\right) \geq p_L V\left(H, b_L\right)$. Hence (IC_H) is satisfied whenever $b_H \leq b_L$ and (IC_L) holds with equality. Thus, the firm's problem becomes

$$\max_{b_H, b_L, b_L \geq b_H \geq 0} \mu_H \cdot \left(\pi\left(H, b_H\right) - W\right) + \mu_L \cdot p_L \cdot \left(\pi\left(L, b_L\right) - W\right) \tag{54}$$

subject to $V\left(L, b_H\right) = p_L \cdot V\left(L, b_L\right)$. Substituting the constraint yields (13).

B.11 Proof of Corollary 3

By Proposition 7, we have $b_H \geq b_L$. Lemmas 1 and 4 imply that $p_L = 1$.

Suppose $b_H \neq b_L$, but (IC_H) holds with strict inequality (and so $r_L = 0$). Since (IC_L) is satisfied with equality, we have $p_H \cdot V\left(L, b_H\right) = V\left(L, b_L\right)$. Since V is log-submodular and $b_H > b_L$, we have $V\left(H, b_H\right) / V\left(L, b_H\right) < V\left(H, b_L\right) / V\left(L, b_L\right)$, and so $p_H \cdot V\left(H, b_H\right) < p_L \cdot V\left(H, b_L\right)$. Hence (IC_H) is not satisfied. Thus, $b_H \neq b_L$ implies (IC_H) holds with equality. In addition, $b_H = b_L$ implies $p_H = p_L$ and $r_L = 0$ from Lemma 4. Hence, regardless of $\left(b_H, b_L\right)$, (IC_H) holds with equality. Substituting (IC_H) and (IC_L) into (6) yields (14).

Moreover, rearranging (IC_H) and (IC_L), yields

$$r_L = \frac{V(H, b_L) \cdot V(L, b_H) - V(L, b_L) \cdot V(H, b_H)}{V(H, b_H) - V(L, b_H)}, \tag{55}$$

which is positive if and only if $b_H > b_L$ given V log-submodular.