Liquidity and Asset Pricing in Incomplete-Market Models

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Three Facts

1. At the household level, consumption does not satisfy an Euler eq.: high MPCs from cash transfers even for wealthy households (Kaplan and Violante [2022]), but low MPC from capital gains (Chodorow-Reich et al. (2021))

2. At the aggregate level, consumption satisfies an Euler equation with the zero-beta rate, the interest rate recovered from equity returns after controlling for risk premia (Di Tella et al. [2023]).

3. But not with the return of safe assets such as Treasury bills (Hansen and Singleton [1982]). There is a large and volatile spread between the zero-beta and safe rates (which is zero in standard models).

Today

- A theory of asset pricing and consumption behavior based on liquidity frictions:
  - safe bonds are "liquid"
  - equities are "illiquid"
- Liquidity frictions can explain household-level consumption behavior such as MPCs (Kaplan and Violante [2022])
- We show that they can also explain why the Euler equation holds for equities but not for safe bonds, as long as PD ratios are volatile and only weakly predict dividend growth
- Consumption CAPM holds: consistent with a flat securities market line
- Takeaway for asset pricing and macro:
  - equity returns: well explained by aggregate consumption (small risk premium)
  - safe asset returns: liquidity premium (consistent with MPCs)

Methodological contribution

- We *analytically* study a two-account general-equilibrium model with aggregate risk
  - We use the canonical two-account setup of Kaplan and Violante [2022]
  - Exact solution is important for asset-pricing results
  - We impose assumptions on production and idiosyncratic risk that permit exact aggregation, while preserving important features we want to study
    - ⋆ building on Krueger and Lustig [2010] and Werning [2015]. Also Auclert [2019], Acharya and Dogra [2020].
  - The solution method involves representing the economy with aggregate shocks in terms of an aggregate-deterministic economy under a stochastic time change

Outline

1. Log economy
2. CRRA economy
3. Quantitative exploration

The Log Utility Model

- Continuum of households $i \in [0,1]$

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \ln C_{it} dt\right]$$

- Technology:

$$\frac{dY_t}{Y_t} = g_t dt + \sigma_Y(Y_t, g_t) \cdot dM_t, \tag{1}$$

$$dg_t = \mu_g(g_t, Y_t)dt + \sigma_g(g_t, Y_t) \cdot dM_t. \tag{2}$$

- Fixed factor shares: α for capital, $1-\alpha$ for labor
- Household i's share of labor income e_{it} follows generator $\mathcal{L}_e$
- Resource constraints:

$$\int_0^1 C_{it} di = C_t = Y_t, \quad \int_0^1 e_{it} di = 1. \tag{3}$$

The Two-Account Budget Constraints

- Budget:

$$dA_{it} = r_{at}A_{it}dt + D_{it}dN_{it} + \sigma_{at}A_{it} \cdot dM_t,$$
$$dB_{it} = \left(r_{bt}B_{it} + e_{it}\left(1-\alpha\right)Y_t - C_{it}\right)dt - (D_{it} + \kappa B_t \mathbb{I}_{D_{it}\neq 0})dN_{it} + \sigma_{bt}B_{it} \cdot dM_t$$

- Notation:
  - A_{it}, B_{it}: illiquid and liquid assets (market value)
  - r_{at}, r_{bt} are expected returns. $\sigma_{at}dM_t$, $\sigma_{bt}dM_t$ are surprise returns
  - $e_{it}\left(1-\alpha\right)Y_t$: labor income (paid into liquid account)
  - D_{it}: purchase of illiquid/sale of liquid conditional on rebalancing
- Trading friction: Poisson arrival χ and fixed cost κ (in units of liquid asset B_t)
- No borrowing: $A_{it} \geq 0$, $B_{it} \geq 0$
- Kaplan and Violante [2022] and Auclert et al. [2023]

Asset Supply

- θ share of the capital income backs liquid assets (no-bubbles)

$$A_t = \mathbb{E}_t \left[\int_t^{\infty} e^{-\int_t^s r_{au} du} (1-\theta)\alpha Y_s ds \right] \tag{4}$$

$$B_t = \mathbb{E}_t \left[\int_t^{\infty} e^{-\int_t^s r_{bu} du} \theta \alpha Y_s ds \right] \tag{5}$$

- P_{at} and P_{bt} are price-dividend ratios

$$P_{at} = \frac{A_t}{(1-\theta)\alpha Y_t}, \quad P_{bt} = \frac{B_t}{\theta \alpha Y_t}$$

State space and generators

- Rescale variables:

$$a_{it} = \frac{A_{it}}{A_t}, \quad b_{it} = \frac{B_{it}}{B_t}, \quad c_{it} = \frac{C_{it}}{Y_t}, \quad d_{it} = \frac{D_{it}}{B_t}$$

- Idiosyncratic state (a_{it}, b_{it}, e_{it}) with generator $\mathcal{L}_{abe}(c_t, d_t; P_{at}, P_{bt})$

- FKE equation describes evolution of measure $\mu_t(a, b, e)$

$$d\mu_t(\cdot) = \mathcal{L}^{\dagger}_{abe}(c_t, d_t; P_{at}, P_{bt})\mu_t(\cdot)dt \tag{6}$$

Equilibrium

Definition

A competitive equilibrium is a set of adapted policy functions $(c_t^*(\cdot), d_t^*(\cdot))$, price processes $(r_{at}^*, \sigma_{at}^*, P_t^{a*}, r_t^{b*}, \sigma_{bt}^*, P_t^{b*})$, and a measure μ_t^* such that (1) policies are optimal in households' problem, (2) μ_t^* satisfies KFE (6) with initial condition $\mu_0^* = \mu_0$, (3) prices and returns satisfy (4) and (5), and (4) markets clear:

$$\begin{aligned} \int b d\mu_t^*(a, b, e) &= 1, \\ \int a d\mu_t^*(a, b, e) &= 1, \\ \int c_t^*(a, b, e) d\mu_t^*(a, b, e) &= 1. \end{aligned} \tag{7}$$

Deterministic Steady State

- Constant aggregate output $Y_t = Y$
- Value function $\bar{V}(a,b,e)$ and policy functions $\bar{c}(a,b,e)$, $\bar{d}(a,b,e)$, PD ratios $\bar{P}_a = \bar{r}_a^{-1}$ and $\bar{P}_b = \bar{r}_b^{-1}$, and measure $\bar{\mu}$
- Assume there exists a steady state equilibrium

Proposition (Log Economy)

Assume $\sigma_Y(Y_t, g_t) = 0$ *and that there exists a steady state equilibrium* $(\bar{c}, \bar{d}, \bar{r}_a, \bar{r}_b, \bar{P}_a, \bar{P}_b, \bar{\mu})$ *with* C^1 *value function* $\bar{V}(a, b, e)$ *satisfying the steady state HJB equation. Then if* $\mu_0 = \bar{\mu}$*, and* $\phi(Y, g) = \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)} \ln(Y_s) ds | Y_t = Y, g_t = g\right]$ *is finite, there exists a competitive equilibrium with aggregate shocks where*

$$c_t^*(\cdot) = \bar{c}(\cdot), \quad d_t^*(\cdot) = \bar{d}(\cdot), \quad P_{at}^* = \bar{P}_a, \quad P_{bt} = \bar{P}_b, \quad \mu_t^* = \bar{\mu}.$$

There is an aggregate consumption Euler equation for both assets

$$r_{jt} = \bar{r}_j + g_t, \tag{8}$$

and assets are locally safe, $\sigma_{at} = \sigma_{bt} = 0$.

What Trickery is This?

- An aggregate Euler (with a different constant in each account) holds
  - despite binding borrowing constraints and uninsured idiosyncratic risk
- Why? an absence of redistribution in response to aggregate shocks
  - Between capital and labor
  - Between liquid and illiquid assets
  - *Present value* of labor income: (1) $\mathcal{L}_e$ invariant to aggregate shocks and (2) interest rates and labor growth move together (EIS $= 1$).
- Reminiscent of results in Krueger and Lustig [2010] and Werning [2015]

The Log Economy is Boring

- The log economy cannot explain our facts:
  - the spread between liquid and illiquid is constant: Euler eq. works for both
  - IES is 1: in the data 1/5 is a better number, and 1 is rejected
  - price-dividend ratios are constant
  - labor income dynamics invariant to the cycle

- Next, consider CRRA economy with $\gamma > 1$:

$$U(C_i) = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \frac{C_{it}^{1-\gamma}}{1-\gamma} dt\right].$$

The CRRA Challenge

- Households with currently low e_{it} (e.g. unemployed) have a backloaded labor income profile relative to those with high e_{it}
- With $\gamma > 1$ interest rates move more than one-to-one with expected consumption growth g_t
- Therefore, lower g_t redistributes from those with high labor income to those with low labor income
    - e.g. the currently unemployed benefit from the onset of a recession. This is crazy, and it breaks aggregation

Solution: cyclical idiosyncratic risk

- Define x_t:

$$x_t = x(Y_t, g_t) = \rho \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)} (Y_s/Y_t)^{1-\gamma} ds | Y_t = Y, g_t = g\right]$$

  - PD ratio in a RA economy, normalized by ρ

- Assume:
  - the generator for labor income risk is $x_t^{-1}\mathcal{L}_e$
  - rebalancing occurs at rate $x_t^{-1}\chi$

- Example: unemployed households are less likely to find a job during a recession and trading frictions become worse

- We are setting up a baseline where aggregate shocks do not have a redistributive effect on impact

Proposition (CRRA Economy)

Assume $\sigma_Y(Y_t, g_t) = 0$ and that there exists a steady state equilibrium of the CRRA economy $(\bar{c}, \bar{d}, \bar{r}_a, \bar{r}_b, \bar{P}_a, \bar{P}_b, \bar{\mu})$ with C^1 value function $\bar{V}(a, b, e)$ satisfying the steady state HJB equation. Then if $\mu_0 = \bar{\mu}$, and $x(Y, g)$ is finite, there exists a competitive equilibrium with aggregate shocks where

$$c_t^*(\cdot) = \bar{c}(\cdot), \quad d_t^*(\cdot) = \bar{d}(\cdot), \quad P_{at}^* = x_t \bar{P}_a, \quad P_{bt}^* = x_t \bar{P}_b, \quad \mu_t^* = \bar{\mu}.$$

Expected asset returns are

$$r_{jt} = \rho + \gamma g_t - \frac{\rho - \bar{r}_j}{x_t}, \tag{9}$$

and asset volatility is $\sigma_{at} = \sigma_{bt} = \frac{\sigma_x(Y_t, g_t)}{x(Y_t, g_t)}$.

Remark: *If we don't start at $\mu_0 = \bar{\mu}$, we can still characterize the full solution using the deterministic path under a stochastic time change: $\tau_t = \int_0^t x_s^{-1} ds$*

Liquidity Premium and PD ratios

- PD ratio x_t and the liquidity premium are now time-varying:

$$s_t = r_{at} - r_{bt} = \mathbb{E}[s_t] \times \frac{x_t^{-1}}{\mathbb{E}[x_t^{-1}]}$$

- When asset prices are high relative to output, the supply of liquidity is high relative to the demand for it, so the liquidity premium is small

- PD ratio has the same behavior as in a RA economy

Risk Premia and the Zero-Beta rate

- Now suppose $\sigma_Y(Y_t, g_t) > 0$ and households can also trade zero-net supply derivatives on dM_t in each account, to dynamically complete the market with respect to aggregate risk

Proposition (Consumption CAPM and Euler equations)

Assume $\bar{V}(a, b, e)$ is concave in a and b, and that the previous regularity conditions hold. Then the price of risk is

$$\pi_j(Y, g) = \gamma \sigma_Y(Y, g).$$

The zero-beta rates satisfy

$$r^0_{jt} = \underbrace{\rho + \gamma g_t - (\gamma + 1)\frac{\gamma}{2}\sigma^2_{Yt}}_{\textit{representative-agent Euler eq.}} - \frac{\rho - \bar{r}^0_j}{x_t}. \tag{10}$$

Quantitative exploration: sufficient statistic

- First obtain a sufficient statistic to characterize returns independently of microeconomic details

$$r^0_{jt} = \mathbb{E}\left[r^0_{jt}\right] + \gamma\left(g_t - \mathbb{E}\left[g_t\right]\right) - \left(\tilde{\rho} - \mathbb{E}\left[r^0_{jt}\right]\right) \times \underbrace{\left(\frac{x_t^{-1}}{\mathbb{E}\left[x_t^{-1}\right]} - 1\right)}_{\approx \log(\hat{d/p})}$$

where $\tilde{\rho} = \rho + \gamma\mathbb{E}[g_t] - (\gamma+1)\frac{\gamma}{2}\sigma_Y^2$ is the "effective impatience rate"

- HA part: summarized in the mean spread $\mathbb{E}[s_t]$

- AP part: summarized in the behavior of $x_t^{-1}/\mathbb{E}[x_t^{-1}]$

Quantitative exploration: Euler for illiquid rate

- Liquid asset: use MZM monetary aggregate ("zero maturity"). It includes cash, checking, saving and money markets. $\mathbb{E}[r^0_{bt}] = -1.5\%$.

- Illiquid asset: stocks. The zero-beta rate from Di Tella et al. [2023] is $\mathbb{E}[r^0_{at}] = 8.5\%$. The mean spread is $\mathbb{E}[s_t] = 10\%$.

- Growth: $\mathbb{E}[g_t] = 1.5\%$, $std(g_t) = 0.5\%$. Consumption growth is not very predictable ($R^2 < 0.17$ assuming a volatility of 1.22%)

- Aggregate Euler for r^0_{at}: $\rho - (\gamma+1)\frac{\gamma}{2}\sigma_Y^2 = 1\%$ and $\gamma = 5$

$$r^0_{at} = \underbrace{\mathbb{E}[r^0_{at}]}_{8.5\%} + \underbrace{\gamma}_{5}(g_t - \mathbb{E}[g_t]) - \underbrace{(\tilde{\rho} - \mathbb{E}[r^0_{at}])}_{=0} \times \left(\frac{x_t^{-1}}{\mathbb{E}[x_t^{-1}]} - 1 \right)$$

Quantitative exploration: Euler for liquid rate

- We want the Euler equation to fail for the liquid rate r^0_{bt}. Consider the linear projection

$$\frac{x_t^{-1}}{\mathbb{E}[x_t^{-1}]} - 1 = \beta(g_t - \mathbb{E}_t[g_t]) + \epsilon_t$$

- Plug it into the expression for r^0_{bt}

$$r^0_{bt} = \mathbb{E}[r^0_{bt}] + (\gamma - \beta\mathbb{E}[s_t]) \times (g_t - \mathbb{E}[g_t]) - \mathbb{E}[s_t] \times \epsilon_t. \tag{11}$$

- We need the log dividend yield to be only weakly predictive of output growth. Which is true (Cochrane (2006)).

Quantitative exploration: log dividend yield

- $std(x_t^{-1}/\mathbb{E}[x_t^{-1}]) = 26\%$ (Campbell and Cochrane [1999]), $\beta = 20$: coefficient in g_t on $x_t^{-1}/\mathbb{E}[x_t^{-1}]$ of 0.0074 (Cochrane (2006))

- Euler eq. works for r_{at}^0 ($R^2 = 1$ by construction) and fails for r_{bt}^0 ($R^2 = 0.28$). Predicting consumption growth with r_{bt} has $R^2 = 0.04$ vs. 0.17 if we use r_{at}.

- Volatility of r_{bt}^0 is $std(r_{bt}) = 2.8\%$, same as in the data

Asset Pricing and Macro

- Aggregate consumption can explain equity returns:
  - Euler equation for the zero-beta rate
  - consumption CAPM (in line with flat securities market line)
- The return of safe assets is the weird thing: it reflects a procyclical but noisy endogenous liquidity premium. Think of safe bonds as close to "money".
- Challenge for macro: rethink role of safe interest rate. Intertemporal substitution vs. liquidity.
- Challenge for asset-pricing: models of volatile PD ratio with small risk premia

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