The Zero Beta Rate

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The Consumption Euler Equation

- The consumption Euler equation,

\[ C_t^{-\sigma} = \mathbb{E}_t \left[ \delta C_{t+1}^{-\sigma} \frac{R_{f,t}}{1 + \pi_{t+1}} \right] \]

is a foundation of modern macro models.

- Problem: it does not describe the data
  - with aggregate consumption \( C_t \), CPI inflation \( \pi_t \), and Treasury bill yield \( R_{f,t} \)
  - Hansen and Singleton [1983], Dunn and Singleton [1986], Yogo [2004]
The Consumption Euler Equation: Theory vs Data

- Formal GMM tests reject: Dunn and Singleton [1986], Yogo [2004]
Responses

- Maybe habits or Epstein-Zin preferences? but see Canzoneri et al. [2007]
- Maybe wrong $C_t$, or time preference ($\delta_t$) shocks?

- Our hypothesis: bonds are convenient, stocks are not
  - Cash and deposits have convenience. We don’t expect the Euler equation to hold with their return. What if bonds are also convenient?

- Implication: all assets without convenience have same risk-adjusted return
  - call this the “zero-beta rate”

- We first estimate the zero-beta rate, then test the Euler equation
Main Result: A Consumption Euler Equation That Works

- cannot reject in formal GMM tests
• Key idea: time-varying estimate of zero-beta rate (Black [1972])
  - Black et al. [1972], Bali et al. [2017] : zero-beta rate 3-8% above Tsy yield
  - could have been problem with CAPM...
  - but Lopez-Lira and Roussanov [2020] and Kim et al. [2021] (among others) find high returns with no factor exposure
  - Some (weak) evidence on time-variation: Black et al. [1972], Shanken [1986]

• Macro models: large, volatile “Euler shock” needed in DSGE models (Smets and Wouters [2007]; Chari et al. [2009]; Fisher [2015])
What We Do

1. Construct a time-varying estimate of the zero beta rate
2. Show that it works in the consumption Euler
3. Implications for monetary policy
The Classic Euler Equation: Three Implications

1. Cross-sectional asset pricing: $0 = \mathbb{E}_t \left[ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{R_{i,t+1} - R_{j,t+1}}{1 + \pi_{t+1}} \right]$

2. Safe bonds: $R_{b,t}^{-1} = \mathbb{E}_t \left[ \delta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1}{1 + \pi_{t+1}} \right]$

3. The zero-beta (covariance w/ sdf) rate: $R_{0,t}^{-1} = \mathbb{E}_t \left[ \delta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1}{1 + \pi_{t+1}} \right]$
The Classic Euler Equation: Three Implications

1. Cross-sectional asset pricing: 
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- (1) is false (cross-sectional AP e.g. Fama and French [1993])
- (2) is false (convenience, Hansen-Singleton)
- idea: test (3) without imposing (1) or (2)
- first: a modified Euler in which (3) but not (1) or (2) holds
Motivating Euler Equation

- Representative household maximizes

$$
\mathbb{E} \left[ \sum_{t=0}^{\infty} \delta^t \xi_t \left( \frac{C_t^{1-\sigma}}{1-\sigma} + \eta(\theta_t) \right) \right]
$$

- $C_t$: consumption, $\theta_t$: asset holdings
- $\eta(\theta_t)$: “convenience” from asset holdings. role: explain convenience yields (2)
- $\xi_t$: exogenous shock to marginal utility, martingale independent of consumption
  - independence from consumption derived from primitive conditions in full model
  - role: explain why consumption doesn’t price the cross-section (1)

- generalized Euler equation for nominal asset return $R_{i,t+1}$:

$$
C_t^{-\sigma} = \frac{\partial \eta(\theta_t)}{\partial \theta_{i,t}} + \mathbb{E}_t \left[ \delta \frac{\xi_{t+1}}{\xi_t} C_{t+1}^{-\sigma} \frac{R_{i,t+1}}{1 + \pi_{t+1}} \right]
$$
The Zero-Beta Rate

- Consider portfolio with (i) no convenience and (ii) uncorrelated with the SDF.
- $R_{t+1}$ vector of $N$ returns, $w \in \mathbb{R}^N$ weights of a zero-beta portfolio
- For zero-beta, zero-convenience portfolios only,
  1. classic consumption Euler holds,

$$C_t^{-\sigma} = \mathbb{E}_t [w' \cdot R_{t+1}] \mathbb{E}_t \left[ \delta \frac{\xi_{t+1}}{\xi_t} \frac{C_{t+1}^{-\sigma}}{1 + \pi_{t+1}} \right] = R_{0,t} \mathbb{E}_t \left[ \delta \frac{C_{t+1}^{-\sigma}}{1 + \pi_{t+1}} \right]$$

  2. expected portfolio return is zero-beta rate:

$$\mathbb{E}_t [w' \cdot (R_{t+1} - R_{0,t})] = 0$$

- Plan: use second + extra structure to construct zero-beta rate, then test first
Factor Structure Implementation

- Implementation: (i) use stocks, and (ii) assume linear factor SDF,

\[ \delta \frac{\xi_{t+1}}{\xi_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{1 + \pi_{t+1}} = (R_{0,t})^{-1} + \sum_{j=1}^{K} \omega_{j,t} (F_{j,t+1} - \mathbb{E}_t [F_{j,t+1}]) + \zeta_{t+1} \]

- \( K \) factors, time-varying prices of risk \( \omega_{j,t}, \zeta_{t+1} \) uncorrelated with returns

- Constant beta of excess returns to factors:

\[ R_{i,t+1} - R_{0,t} = \alpha_i + \sum_{j=1}^{K} \beta_{ij} F_{j,t+1} + \epsilon_{i,t+1}, \mathbb{E}_t [F_{j,t+1} \epsilon_{i,t+1}] = 0 \]

- note: \( R_{0,t} \), not \( R_{b,t} \), defines excess returns

- Zero-beta spread vs Tsy yield ("convenience spread") affine in \( L \) instruments \( Z_t \):

\[ R_{0,t} = R_{f,t} + \gamma' \cdot Z_t \]

- \( Z_{0,t} = 1 \); extension: \( \beta_{ij,t} \) linear in \( Z_t \) (ala "conditional CAPM")

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Portfolio Interpretation

- Pretend we know excess returns $R_{t+1} - R_{0,t}$

1. Regress excess returns on factors to get betas

2. Form minimum variance zero-beta portfolio, $w^*(\gamma, \beta)$
   - minimum variance for efficiency, Ledoit and Wolf [2017] for robustness

3. Predict returns of portfolio using instruments $Z_t$,

   $$w^*(\gamma, \beta)' \cdot R_{t+1} - R_{f,t} = \gamma' \cdot Z_t + \kappa_{t+1}$$
Portfolio Interpretation

1. Regress excess returns on factors to get betas
   - moments \( \mathbb{E}[F_{j,t+1}\epsilon_{i,t+1}], F_{0,t+1} = 1 \)
2. Form minimum variance zero-beta portfolio, \( w^*(\gamma, \beta) \)
   - minimum variance for efficiency, Ledoit and Wolf [2017] for robustness
3. Predict returns of portfolio using instruments \( Z_t \),
   \[
   w^*(\gamma, \beta)' \cdot R_{t+1} - R_{f,t} = \gamma' \cdot Z_t + \kappa_{t+1}
   \]
   - moments \( \mathbb{E}[\kappa_{t+1}Z_t] = 0 \)
   - Feasible: both moments at same time with GMM
     - inspired by Shanken [1986] MLE procedure
     - if all factors tradable: non-linear least squares w/ GLS
GMM Moments

- Let $\theta = (\alpha, \beta, \gamma)$ be the relevant parameters of the model.
- Define the orthogonal projection matrix, $H(\beta) = I - \beta \beta^+$.
  - If $w \in \mathbb{R}^N$ are portfolio weights, $\hat{w} = H(\beta) \cdot w$ are portfolio weights with zero beta.
- Time-series moments $(\alpha, \beta)$ + instrumented asset pricing moments $(\gamma)$:

$$\begin{align*}
g_{t+1}(\theta) &= \begin{bmatrix} \epsilon_{t+1}(\theta) \otimes F_{t+1} \\ H(\beta) \cdot (R_{t+1} - R_{f,t} - \gamma' \cdot Z_t) \otimes Z_t \end{bmatrix}
\end{align*}$$

- Weight second group by $w^*(\gamma, \beta) = H(\beta)w^*(\gamma, \beta)$ for exact identification,

$$W(\theta) = \begin{bmatrix} I & 0 \\ 0 & w^*(\gamma, \beta)w^*(\gamma, \beta)' \otimes I_L \end{bmatrix}$$
Data

- Stock portfolios: size by value by market beta sorted portfolios + industries
- Factors: five equity factors (Fama and French [2015]) + 2 bond factors (Fama and French [1993]). Also: consumption SDF (doesn’t matter)
- Instruments: t-bill yield, 12m trailing inflation, unemployment, term spread, excess bond premium (EBP)
- Consumption: real Non-Durable + Services per capita
- Reasoning + Robustness in paper
Table 1: Predicting the Zero-Beta Rate

<table>
<thead>
<tr>
<th></th>
<th>(1) GMM</th>
<th>(2) OLS (inf.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lrf</td>
<td>1.186 (0.914)</td>
<td>1.187 (0.789)</td>
</tr>
<tr>
<td>Lump</td>
<td>0.105 (0.0986)</td>
<td>0.105 (0.0965)</td>
</tr>
<tr>
<td>Lebp</td>
<td>-0.603 (0.342)</td>
<td>-0.603 (0.309)</td>
</tr>
<tr>
<td>Ltsp</td>
<td>0.310 (0.118)</td>
<td>0.310 (0.119)</td>
</tr>
<tr>
<td>L2cpi_rolling</td>
<td>-2.582 (1.175)</td>
<td>-2.586 (1.048)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.718 (0.137)</td>
<td>0.716 (0.134)</td>
</tr>
<tr>
<td>Wald/F</td>
<td>21.46</td>
<td>5.012</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000663</td>
<td>0.000167</td>
</tr>
<tr>
<td>Observations</td>
<td>574</td>
<td>574</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.
Discussion

- Significant predictability (Wald/F); minimum-variance helps here
- High constant: 0.7%/mo excess return (not too surprising)
  - 3.2% std. dev., 0.8 annual Sharpe ratio vs. T-bills
- Spread increasing in rate level (Nagel [2016]), statistically weak
- Spread decreasing in inflation (contra Cohen et al. [2005] story?)
- Inverted TS, high EBP, low ump: bad expected returns when recession soon
  - Macro variables help predict these stock returns
Our perspective: $R_{0,t} - R_{f,t} = \mathbb{E}_t [R_{p,t+1} - R_{f,t}]$ represents convenience yield

Alternative perspective: $R_{p,t+1} - R_{f,t}$ is an omitted factor
- with a high Sharpe ratio, uncorrelated with all other factors
- by no-arbitrage, there is an SDF the prices the stocks + Treasury bills

The two perspectives can co-exist within the same model
- Frazzini and Pedersen [2014]
- and with other stories: Hong and Sraer [2016], Bali et al. [2017]

Our perspective can explain why $R_{0,t}$ predicts consumption growth
The Linearized Euler Equation

- Linearizing the consumption Euler equation:
  \[ E_t[\Delta c_{t+1}] = \sigma^{-1} \ln(\delta) + \sigma^{-1} (r_{0,t} - E_t[\pi_{t+1}]) \]

- our figures line up the means, scale by standard deviations
- in effect, choosing \( \delta \) using the means and \( \sigma \) using the std. devs.

- Next:
  1. revisit figures
  2. compare \( r_{0,t} \) and \( r_{f,t} \) as predictors of \( \Delta c_{t+1} \)
  3. discuss weak identification problem
  4. conduct weak-i.d.-robust GMM inference
Main Result: A Consumption Euler Equation That Works

- Predictive regressions for inflation and consumption growth using $Z_t$
• No visually detectable differences when omitting consumption factor
Robustness: Ridge Regressions

- $\gamma$ and consumption prediction penalized using ridge, 10-fold cross-validation
What Can Go Wrong

- Too many factors: estimates noisy but unbiased
- Omitted factors:
  - omitted factor with constant risk price: only level biased, Euler still works
  - omitted factor with one-month ahead return predictability by our instruments: bias
- Too many instruments: weak identification (discussed next)
- Not enough instruments:
  - need at least two
  - bias if omitted instrument predicts either consumption growth or portfolio returns
Placebo: 6-11y Treasury Bond Returns

- convenience (bonds) and predictable risk premium [Campbell and Shiller, 1991]

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Recap

- Two predictive regressions:

\[ \Delta c_{t+1} = \sigma^{-1} \ln(\delta) + (\sigma^{-1} \gamma^c)' \cdot Z_t + \epsilon_{t+1}^c, \]

\[ r_{p,t+1} - \pi_{t+1} = (e_b + \gamma - \gamma^\pi)' \cdot Z_t + \epsilon_{t+1}^0. \]

- Define \( \hat{\gamma} = \sigma^{-1} \gamma^c - e_b + \gamma - \gamma^\pi \)

- Our graphs show \( \hat{\gamma}' \cdot E[Z_tZ'_t] \cdot \hat{\gamma} \) is small (point estimates)

- Next steps:
  1. Test statistically if non-linear Euler can be rejected
     1.1 challenge: potential for weak instruments
  2. Test economically: do monetary shocks affect \( \hat{\gamma}' \cdot Z_t \) (at point estimates)?
• Big picture: Stock and Wright [2000] meets Cochrane [2009]
  1. Conjecture value of $\sigma_0$ (null hypothesis)
  2. Estimate $\hat{\theta}(\sigma_0)$ using previous procedure
     2.1 constructs same zero-beta rate given $\sigma_0$
  3. Estimate $\hat{\delta}(\sigma_0)$ using $\mathbb{E}[\delta(\frac{C_{t+1}}{C_t})^{-\sigma_0} \frac{1}{1+\pi_{t+1}} R_{0,t}(\gamma)] = 1$
  4. Test using unused moments $\mathbb{E}[(\delta(\frac{C_{t+1}}{C_t})^{-\sigma_0} \frac{1}{1+\pi_{t+1}} R_{0,t}(\gamma) - 1)Z_{l,t}] = 0$
• S-set: values of $\sigma_0$ not rejected with 95% confidence
• OLS moments ($\alpha, \beta$) + asset pricing moments ($\gamma$) + cons. Euler ($\delta$):

$$
\begin{align*}
  g_{t+1}(\theta, \delta, \sigma_0) &= \\
  &= \left[ \begin{array}{c}
  \epsilon_{t+1}(\theta) \otimes F_{t+1}(\sigma_0) \\
  H(\beta) \cdot (R_{t+1} - R_{0,t}(\gamma)) \otimes Z_t \\
  \left( \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma_0} \frac{1}{1 + \pi_{t+1}} R_{0,t}(\gamma) - 1 \right) \otimes Z_t 
  \end{array} \right]
\end{align*}
$$

• Weight matrix:

$$
W(\theta) = \left[ \begin{array}{cccc}
  I & 0 & 0 \\
  0 & w^*(\gamma, \beta) w^*(\gamma, \beta)' \otimes I_L & 0 \\
  0 & 0 & e_0 e_0' 
\end{array} \right]
$$

• Same exact identification scheme for ($\alpha, \beta, \gamma$)
  - will recover same zero-beta rate given $\sigma_0$

• Exactly identify $\delta$ by average cons. Euler
Testing with Unused Moments

- For $l > 0$, the unused moments are

$$g_{l,t+1}(\theta, \delta, \sigma_0) = \left( \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma_0} \frac{1}{1 + \pi_{t+1}} R_{0,t}(\gamma) - 1 \right) Z_{l,t}$$

- Let $\psi_{Test}(\sigma_0)$ be the vector $\frac{1}{T} \sum_{t=1}^{T} g_{l,t}(\hat{\theta}_1(\sigma_0), \hat{\delta}(\sigma_0), \sigma_0)$

- Let $\hat{V}_{Test}(\sigma_0)$ be the (robust) covariance matrix of $\psi_{Test}(\sigma_0)$

- Following Stock and Wright [2000]: under null of $\sigma = \sigma_0$,

$$\hat{S}(\sigma_0) = \psi_{Test}(\sigma_0)' \cdot \hat{V}_{Test}(\sigma_0)^{-1} \cdot \psi_{Test}(\sigma_0) \to^d \chi^2_L$$

  - robust to $\sigma_0$ weak i.d., not most powerful (Andrews [2016])

- Also show results for $R_{f,t}$ and $R_{m,t+1}$ in place of $R_{0,t}$ (Yogo [2004])
S-Set Results

- \( R_{f,t} \): rejected
- \( R_{m,t+1} \): not identified
- \( R_{0,t} \): reject \( \sigma \leq 1.5 \), not reject \( \sigma \geq 1.5 \)

- Nothing can reject for \( \sigma \geq 20 \) (COVID, rare disaster)

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• The consumption Euler equation holds when applied to the zero-beta rate
  • in contrast to using a Treasury bill rate (rejected) or the market return (unidentified)
• Robustness:
  • Test assets: More sorts
  • Factors: linear cons., Mkt+cons., FF3+cons., linear betas
  • Instruments: +shadow spread, +lag cons., +CAPE, BAAS instead of EBP
  • Others: Non-durable goods cons. only, pre-COVID
When recessions are imminent (inverted term structure, high credit spreads, but currently low unemployment), agents expect:

1. negative consumption growth (generates desire to save)
2. low risk-adjusted (zero-beta) stock returns (offsets desire to save)

Interest rates don’t enter this calculation

- short-dated bonds are held for convenience
- longer-dated bonds inherit some convenience via financing

Natural question: how does monetary policy change the zero-beta rate?
Monetary Shocks and the Zero-Beta Rate

- Is the convenience yield endogenous (concern of Chari et al. [2009])?
- Tension:
  - fed funds hike raise rates more generally
  - but lower consumption growth
  - inconsistent with standard Euler equation
- Suppose $R_{0,t} = \gamma' \cdot Z_t$ is structural
- How do Nakamura and Steinsson [2018] shocks affect $\gamma' \cdot Z_t$?
  - updated shocks from Acosta [2022]
  - paper: Romer and Romer [2004] shocks
Effects of NS Shocks

- change from $t-1$ to $t+h$ regressed on NS shock in month $t$
- rates scaled ($1 = 1:1$ with fed funds)

Graph showing the effects of the Nakamura-Steinsson shock over time.
**Interpretation**

- In response to a surprise monetary hike:
  - Data: consumption growth falls, then (maybe) rises (“hump”)
  - Vanilla NK: consumption drops on impact, then grows
  - standard fix: habits
  - but habits don’t fix Euler (Canzoneri et al. [2007]), inconsistent with MPCs (Auclert et al. [2020])

- Our story: zero-beta rate falls on impact, cons. gr. falls, vanilla Euler works
  - standard errors too large to test reversion (second part of “hump”)
  - alternative to sticky information hypothesis (Auclert et al. [2020])
Related Papers on Stock/Bond Segmentation

- Itskhoki and Mukhin [2021] exchange rate disconnect

- ROE on arbitrages (say, JPY-USD CIP) is 3-7% over bills (Boyarchenko et al. [2018])

- High return on physical capital: Gomme et al. [2011], Farhi and Gourio [2018]

- Beta anomaly (Frazzini and Pedersen [2014], Hong and Sraer [2016])

- Corporate finance implications thereof (Baker and Wurgler [2015], Baker et al. [2020])

- Equity premium puzzle (Bansal and Coleman [1996])
Conclusion

• The intertemporal price of consumption is not the yield on a Treasury
• The consumption Euler works— if you use the zero-beta rate
• This changes our understanding of monetary policy:
  • monetary shocks substantially alter convenience yields
• 3x3x3 beta by size by \{value, prof., inv.\} + 49 industry portfolios
No Consumption Factor

- No consumption-related factor
Linear Consumption Factor

- Linear consumption factor + separate inflation factor
Market Factor

- Market + Non-Linear Consumption factor only

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FF3 Factors

- Market, Size, Value, and Non-Linear Consumption factors only

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Linear Betas

- $\beta_t = \beta_0 + \beta_1 \cdot Z_t$; 37 factors (6 factors $\times$ 6 $Z$ + 1 consumption-related)

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With Shadow Spread Instrument

- Includes Lenel et al. [2019] bill vs. term-structure-extrapolated bill as instrument

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With Lagged Consumption Instrument

- Includes $\Delta c_{t-1}$ as instrument

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With CAPE Instrument

- Includes Campbell-Shiller cyclically-adjusted P/E ratio as instrument
With BAA-Tsy in place of EBP

- Includes Moody’s BAA-Treasury spread instead of EBP as instrument

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With Non-Durable Goods Consumption Only

- Consumption is real non-durable goods consumption per capita

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Without COVID

- Data sample ends in December 2019

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